PRIMARY ROUND 3 solutions

P3.1.

Whilst visiting their grandmother, the MacDonald children found an old diary where they found an entry:
‘On New Year's Eve, each of our five children resolved that they would, from New Year's Day, get down in time for breakfast or pay fines as follows:
Ailsa, the youngest was to pay 1 penny; Basil 1½ pennies; Charles 2 pennies; Dorothy 2½ pennies and Elgar, the eldest, 4 pennies for each late appearance.’
A few pages later they found this:
‘Three of them were late 3 times during the month and the other two of them 4 times. The total fines amounted to an exact number of shillings.’
What did each child pay in fines during the month?
In those days there were 12 pennies in 1 shilling.

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P3.2.

At a school, 15 students were absent on Monday, 12 were absent on Tuesday and 9 were absent on Wednesday. If none of the students was absent on all three days, what is the smallest possible total number of students that were absent on at least one day?
Justify your answer.

Solution

The total number of absences is $15 + 12 + 9 = 36$. If no student was absent on all three days then each student was absent on at most two of the days. Therefore there must be at least $\frac{36}{2} = 18$ students involved.

We need to check that it is possible to make the daily totals with just 18 students.

If there are 9 students absent on only Monday and Tuesday, 6 students absent on only Monday and Wednesday, and 3 students absent on only Tuesday and Wednesday then the total number of students involved is $9 + 6 + 3 = 18$ and the totals for each day are correct:

Absences	Monday	9	6		= 15
	Tuesday	9		3	= 12
	Wednesday		6	3	= 9

So the smallest possible total number of students that were absent on at least one day is 18.

(Note that it is necessary to show both that 18 students is the minimum possible number and that it can actually be achieved.)

P3.3.

The directors of a company which specialises in the construction of cubes are planning to build a car park at the front of their building. This car park is in the shape of a rectangle, with a total area of 3055 square metres. They make a request to the builders that the car park is made up of square slabs, all of different sizes, and have calculated that it can be done using squares of side 3, 5, 6, 11, 17, 19, 22, 23, 24 and 25 metres.

What must be the dimensions of the car park? How can the slabs be placed to fit?

Solution

$$3055 = 5 \times 13 \times 47$$

Dimensions cannot have length 5 or 13 (slabs too large) so the car park must be 65×47.

The largest slabs will be hardest to place, so the 4 largest should go at the corners.

$25 + 22 = 24 + 23 = 47$, so the 25 and 22 squares should touch and the 24 and 23 should touch.

The next largest squares are 19 and 17. We would like to place these in the middle of the long sides. $65 - 19 = 46 = 22 + 24$, so place the 19 square between 22 and 24.

$65 - 17 = 48 = 23 + 25$, so place the 17 square between 23 and 25.

Then put the small squares in the space in the middle,

Final construction