Junior Division: Problems 2

J1.

A cruise liner, which is 100m long, is sailing at 20 km/hr. As its bow passes a buoy, a passenger starts to walk from bow to stern at 4 km/hr. When he reaches the stern, how far past the buoy will he be?

Solution 1

He is walking at 4 km/hr which is 4000/60 metres/minute. So to walk 100 metres takes 100/(4000/60) = 3/2 minutes. In 3/2 minutes the liner travels 20000/60 × 3/2 metres = 500 metres. As the liner is 100 metres long and he is now at the stern, he is 400 m past the buoy.

Solution 2

For the man to walk 100 m = 0.1 km at 4 km/hr takes $\frac{0.1}{4}$ hr. In this time the bow of the liner travelling at 20 km/hr moves $20 \times \frac{0.1}{4}$ km $= 0.5$ km $= 500$ m. The man is at the stern of the liner, 100 m behind the bow, and so he has moved 500 – 100 = 400 m past the buoy.

J2.

The directors of a company which specialises in the construction of cubes are planning to build a car park at the front of their building. This car park is in the shape of a rectangle, with a total area of 3055 square metres. They make a request to the builders that the car park is made up of square slabs, all of different sizes, and have calculated that it can be done using squares of side 3, 5, 6, 11, 17, 19, 22, 23, 24 and 25 metres.

What must be the dimensions of the car park? How can the slabs be placed to fit?

Solution

$$3055 = 5 \times 13 \times 47$$

Dimensions cannot have length 5 or 13 (slabs too large) so the car park must be 65×47.

The largest slabs will be hardest to place, so the 4 largest should go at the corners.

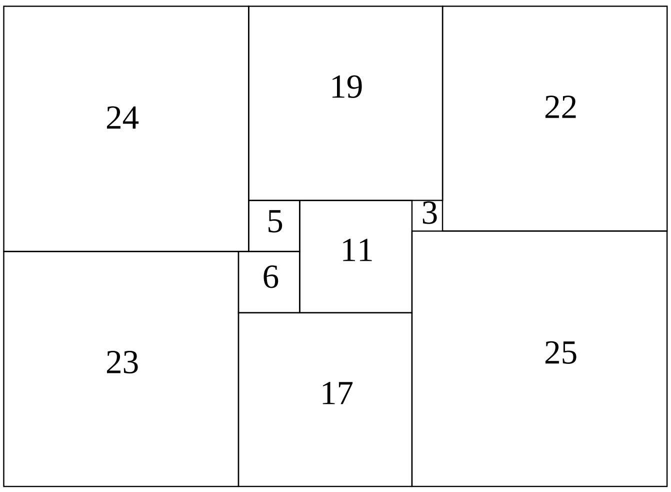
25 + 22 = 24 + 23 = 47, so the 25 and 22 squares should touch and the 24 and 23 should touch.

The next largest squares are 19 and 17. We would like to place these in the middle of the long sides. $65 - 19 = 46 = 22 + 24$, so place the 19 square between 22 and 24.

$65 - 17 = 48 = 23 + 25$, so place the 17 square between 23 and 25.

Then put the small squares in the space in the middle,

Final construction



J3.

At a school, 15 students were absent on Monday, 12 were absent on Tuesday and 9 were absent on Wednesday. If none of the students was absent on all three days, what is the smallest possible total number of students that were absent on at least one day?

Solution 1

The total number of absences is $15 + 12 + 9 = 36$. If no student was absent on all three days then each student was absent on at most two of the days. Therefore there must be at least $\frac{36}{2} = 18$ students involved.

We need to check that it is possible to make the daily totals with just 18 students.

If there are 9 students absent on only Monday and Tuesday, 6 students absent on only Monday and Wednesday, and 3 students absent on only Tuesday and Wednesday then the total number of students involved is $9 + 6 + 3 = 18$ and the totals for each day are correct:

Absences	Monday	9	6		= 15
	Tuesday	9		3	= 12
	Wednesday		6	3	= 9

So the smallest possible total number of students that were absent on at least one day is 18.

(Note that it is necessary to show both that 18 students is the minimum possible number and that it can actually be achieved.)

Solution 2

There are a total of 15 + 12 + 9 = 36 absences. If no student was absent on all three days, then each was absent for at most two of the days. Therefore there must be at least $\frac{36}{2} = 18$ students involved.

Can we make the daily totals given with just 18 students?

Let a be the number of students absent on Monday and Tuesday.

Let b be the number of students absent on Monday and Wednesday.

Let c be the number of students absent on Tuesday and Wednesday.

Then $\quad a + b = 15 \quad a + c = 12 \quad b + c = 9$

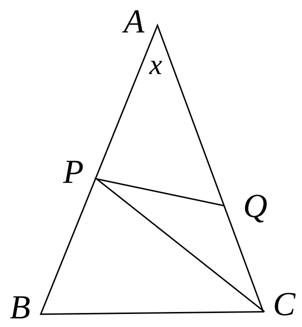
Hence $a = 9$, $b = 6$ and $c = 3$.

So we can make the daily totals required with just 18 students with each absent on two days as follows:

	Mon	Tues	Wed
	9	9	
	6		6
		3	3
total	15	12	9

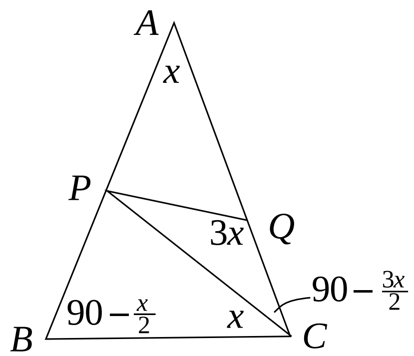
Note that it is necessary to show both that 18 students is the minimum possible number and that it can actually be achieved – solving the equations is not needed, but it does also show that the solution is unique.

J4.



Triangle ABC has $AB = AC$ and $\angle BAC = x$ is less than $60°$. Point P lies on AB such that $CB = CP$. Point Q lies on AC such that $CQ = PQ$.
Determine $\angle CQP$ in terms of x.

Solution



$$\angle ABC = \angle ACB = \tfrac{1}{2}(180 - x) = 90 - \tfrac{1}{2}x$$

(base angles of isosceles triangle ABC equal)

Triangles ABC and CBP are similar (both isosceles with same base angle at B)
So $\angle BCP = \angle BAC = x$.

$$\angle QCP = \angle ACB - \angle BCP = \left(90 - \tfrac{1}{2}x\right) - x = 90 - \tfrac{3}{2}x.$$

Triangle CQP is isosceles, so its apex angle $\angle CQP$ is

$$180 - 2 \times \text{base angle} = 180 - 2\left(90 - \tfrac{3}{2}x\right) = 3x.$$

J5.

One hundred years ago there was a gathering to present an award to a local teacher in recognition of many years of service.

The women there numbered four-fifths of the men, 40% of whom were unmarried. Half of the married women were accompanied by their husbands and a quarter of the married men by their wives. Thirty of the women were unmarried.

How many people were there at the gathering?

Solution

If the number of men is m then

	Men (m)	Women
Married	$\frac{3}{5}m$	$\frac{4}{5}m - 30$
Single	$\frac{2}{5}m$	30
Total	m	$\frac{4}{5}m$

However, half the married women were with their husbands which must be the same number as quarter of the married men who came with their wives.

$$\frac{2}{5}m - 15 = \frac{3}{20}m$$
$$\frac{1}{4}m = 15$$
$$m = 60$$

There were 60 men and 48 women. There were 108 people at the gathering.