



# The Scottish Mathematical Council

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## MATHEMATICAL CHALLENGE 2024–2025

### Primary Division: Problems III

### SOLUTIONS

#### P3.1

A group of children were playing a ring toss game. On their turn, each player tossed three rings at the three pegs, and every ring tossed landed on a peg. Sometimes there were two or three rings on the same peg. A ring over peg A is worth one point, over peg B three points and over peg C five points. Surprisingly, every child in the group had a different total score.

What is the largest possible number of children in the group?

#### *Solution 1*

Possible throws are

111	113	115	
133	135	155	
333	335	355	555

Corresponding totals are

3	5	7	
7	9	11	
9	11	13	15

There are 7 different totals, and so the largest possible number of children in the group is 7.

#### *Solution 2*

The smallest possible total is  $1 + 1 + 1 = 3$ .

The largest possible total is  $5 + 5 + 5 = 15$ .

The sum of 3 odd numbers is always odd, and so the total score is always odd.

All odd scores between 3 and 15 are achievable:

$$5 = 3 + 1 + 1 \quad 7 = 3 + 3 + 1 \quad 9 = 3 + 3 + 3 \quad 11 = 5 + 5 + 1 \quad 13 = 5 + 5 + 3$$

There are 7 different totals, and so the largest possible number of children in the group is 7.

### P3.2

A triangle can be formed with sides of lengths 3, 4 and 6 cm but not with sides of lengths 3, 4 and 7 cm. Oliver has 8 sticks each with length a whole number of cm, but he cannot form a triangle with any 3 of them.

What is the shortest possible length of the longest stick?

#### *Solution*

Start with the shortest possible two sticks both of length 1 cm.

There cannot be a third 1 cm stick because then they could form a triangle. So add a 2 cm stick, which cannot form a triangle. Now take the two longest sticks and add their lengths to get the length in cm of the next stick:

1, 1, 2, 3, 5, 8, 13, 21

So the shortest possible length of the longest stick is 21 cm.

### P3.3

In a chemistry lab there are two bottles each containing a mixture of acid and water:

bottle A contains 140 grams of which 10% is acid,

bottle B contains 60 grams of which 25% is acid.

The lab technician uses some of the mixture from each of the bottles to create a mixture with mass 120 grams of which 15% is acid. Then the lab technician mixes the remaining contents of the bottles to create a new mixture. What percentage of the new mixture is acid?

#### *Solution*

Note that there is no need to find how much of the contents of each bottle is used to create the first mixture, as long as we know that it is possible.

Bottle A: 140 g total, 10% acid = 10% of 140 g = 14 g

Bottle B: 60 g total, 25% acid = 25% of 60 g = 15 g

A and B together: 140 + 60 = 200 g total 14 + 15 = 29 g acid

First mixture: 120 g total, 15% acid = 15% of 120 g = 18 g

Amounts remaining: 200 – 120 = 80 g total, 29 – 18 = 11 g acid

The new mixture, created from the amounts remaining, is  $\frac{11}{80} = 13.75\%$  acid.