PRIMARY ROUND 1 solutions

P1.1.

James and Mia play a two-person game in which the winner gains 2 points and the loser loses 1 point. Yesterday James won exactly 4 games and Mia had a final score of 6 points.

What was James' final score?

Solution

The four games James won give him a total score of $2 \times 4 = 8$ points, and Mia a total score of $4 \times (-1) = -4$ points.

To make Mia's final score up to 6 points, she needs an increase of $6 - (-4) = 10$ points, Each win scores 2 points, so she needs $10 \div 2 = 5$ wins. But for his 5 losses James will score $5 \times (-1) = -5$ points. So his final score was $8 + (-5) = 3$ points.

P1.2.

Over coffee the other day my friend MacAngus and I were mulling over the passing of time. ‘Four years ago’, MacAngus said, ‘I was four times as old as young Callum and now I'm only three times as old.’

‘If you go on like that’ I said, ‘you’ll soon both be the same age.’

When we finished laughing over this fallacy, it occurred to me that from what MacAngus said it would be possible to figure out his age. How old is my friend MacAngus?

Solution 1

Let the age now of MacAngus be m years and the age now of young Callum be c years.

Four years ago:

$$m - 4 = 4(c - 4)$$

And now:

$$m = 3c$$

So $3c - 4 = 4c - 16$ hence $c = 12$.

So MacAngus is 36 years old.

Guidelines

1 mark for the age relation four years ago (using symbols or words)

1 mark for the age relation now (using symbols or words)

1 mark for obtaining the age of Callum

1 mark for obtaining the present age of MacAngus

Solution 2

Using a table of possible ages:

Age now		Age 4 years ago		
Callum	MacAngus	Callun	MacAngus	
5	15	1	11	
6	18	2	14	
7	21	3	17	
8	24	4	20	
9	27	5	23	
10	30	6	26	
11	33	7	29	
12	36	8	32	See Note 1
13	39	9	35	See Note 2

Note 1: 4 years ago, MacAngus was 4 times as old as Callum.

Note 2: For older ages, MacAngus is aways less than 4 times as old as Callum.

Answer: So MacAngus is now 36 years old.

P1.3.

Four explorers wish to get one of their number as far as possible into the wilderness from their base. Each explorer can carry supplies for up to 10 days. At any time supplies can be transferred between explorers and individual explorers can return to base, provided they have sufficient supplies for the return journey. Supplies cannot be left unattended in the wilderness.

What is the greatest number of days the lead explorer can travel from the base so that all return safely?

Solution 1

The 4 explorers must set off together, otherwise they will not be able to transfer supplies.

General rule: Each time an explorer returns to base they should retain just enough supplies to reach the base and leave each of the remaining explorers with a full load of supplies.

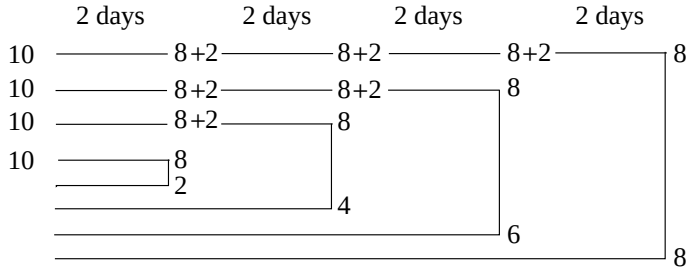
So the first explorer returns after 2 days, taking 2 days of supplies for themself and topping up the other 3 with 2 days of supplies each, taking their loads back to 10 days supplies.

Then the second explorer returns after 4 days, taking 4 days of supplies for themself and topping up the other 2 with 2 days of supplies each, taking their loads back to 10 days supplies.

The third explorer returns after 6 days, taking 6 days of supplies for themself and topping up the lead explorer with 2 days of supplies, taking their load back to 10 days supplies.

The lead explorer is now 6 days out with 10 days of supplies, so they can travel another 2 days out and still have 8 days supplies to get back to base.

This can be shown in a diagram:



The greatest number of days the lead explorer can travel from base is 8 days.

Note: If the lead explorer trusts their fellow explorers to meet them in the wilderness with more supplies they could get even further!

Solution 2

Each time an explorer returns to base they should have retained just enough supplies to reach the base and leave each of the remaining explorers with a full load of supplies.

Let x be the number of days or extra number of days before each man leaves to return.

First man uses $2x$ supplies.

Second man uses $4x$ supplies.

Third man uses $6x$ supplies.

Last man uses $8x$ supplies.

So

$$\begin{aligned} 8x + 6x + 4x + 2x &= 40 \\ 20x &= 40 \\ x &= 2 \end{aligned}$$

The greatest number of days the lead explorer can travel from base is 8 days.

So the first explorer must return after 2 days, taking 2 days of supplies and topping up the others with 2 days of supplies each.