Junior Division: Problems 2

J1.

A triangle can be formed with sides of lengths 3, 4 and 6 cm but not with sides of lengths 3, 4 and 7 cm. Oliver has 8 sticks each with length a whole number of cm, but he cannot form a triangle with any 3 of them.

What is the shortest possible length of the longest stick?

Solution

Start with the shortest possible two sticks both of length 1 cm.

There cannot be a third 1 cm stick because then they could form a triangle. So add a 2 cm stick, which cannot form a triangle. Now take the two longest sticks and add their lengths to get the length in cm of the next stick:

1, 1, 2, 3, 5, 8, 13, 21

So the shortest possible length of the longest stick is 21 cm.

J2.

In a chemistry lab there are two bottles, each containing a mixture of acid and water:

bottle A contains 140 grams of which 10% is acid,

bottle B contains 60 grams of which 25% is acid.

The lab technician uses some of the mixture from each of the bottles to create a mixture with mass 120 grams of which 15% is acid. Then the lab technician mixes the remaining contents of the bottles to create a new mixture. What percentage of the new mixture is acid?

Solution

Note that there is no need to find how much of the contents of each bottle is used to create the first mixture, as long as we know that it is possible.

Bottle A: 140 g total, 10% acid = 10% of 140 g = 14 g Bottle B: 60 g total, 25% acid = 25% of 60 g = 15 g A and B together: 140 + 60 = 200 g total 14 + 15 = 29 g acid First mixture: 120 g total, 15% acid = 15% of 120 g = 18 g Amounts remaining: 200 - 120 = 80 g total, 29 - 18 = 11 g acid

The new mixture, created from the amounts remaining, is $\frac{11}{80} = 13.75\%$ acid.

J3.

Three types of item, A, B and C, are for sale. Items of type A sell at 8 for £1. Items of type B sell for £1 each. Items of type C sell for £10 each. A selection of 100 items of all three types costs £100. How many items of type B were there in the selection?

Solution

So

Let a be the number of items of type A in the selection. Let b be the number of items of type B in the selection. Let c be the number of items of type C in the selection.

$$\frac{a}{8} + b + 10c = 100$$

 $a + b + c = 100$
 $b = 100 - a - c$

$$\frac{a}{8} + 100 - a - c + 10c = 100$$
$$-\frac{7}{8}a + 9c = 0$$
$$7a = 72c$$

Either a = c = 0, which would mean that there were item types missing from the selection, and hence is not possible.

Or a = 72 and c = 7 (larger multiples of these are not possible as there are only 100 items in all.)

Hence b = 100 - 72 - 7 = 21i.e. there are 21 items of type B.

J4.

A bag contains 21 balls, each of which is red or blue. The balls are identical except for their colour. Sasha reaches into the bag and removes two balls at random. Each ball in the bag is equally likely to be removed. The probability that two red balls are removed is exactly $\frac{1}{2}$.

How many of the 21 balls are red?

Solution

Let the number of red balls in the bag be *r*.

P(choosing first red) =
$$\frac{r}{21}$$
 and P(choosing second red) = $\frac{r-1}{20}$.
The probability that 2 red balls are removed is $\frac{r(r-1)}{21 \times 20}$, which must equal $\frac{1}{2}$.
 $r(r-1) = \frac{1}{2} \times 21 \times 20 = 21 \times 10 = 15 \times 14$
 $r = 15$

So there are 15 red balls in the bag at the start.

J5.

Two joggers live beside a canal. The distance between their houses along the towpath is 5 miles. They each set out at the same time to jog along the towpath to the other's house and back. One jogs at a constant speed of 5 mph and the other is faster with a constant speed of 7 mph.

How far from home will the faster jogger be when they meet for the second time? And how long after they set out is this?

Solution

Let the slower jogger jog at 5 mph from his home at A and the faster jogger jog at 7 mph from his home at B.

The slower jogger will take 1 hour or 60 minutes each way. The faster jogger will take 5/7 hour or approximately 43 minutes each way.



The joggers start 5 miles apart and they close at a speed of 5 + 7 = 12 mph. So they first meet after 5/12 hours or 25 minutes.

From the diagram you can see that their second meeting will occur as if they had started 15 miles apart, still closing at 12 mph. So their second meeting is after 15/12 = 5/4 hours, or 75 minutes.

The faster jogger will have travelled 7 $\times \frac{5}{4} = 8\frac{3}{4}$ miles, $1\frac{1}{4}$ miles from B.

Check: The slower jogger will have travelled $5 \times \frac{5}{4} = 6\frac{1}{4}$ miles, and so will also be $1\frac{1}{4}$ miles from B.

So at their second meeting the faster jogger will be $1\frac{1}{4}$ miles from home and this is 1 hour 15 minutes after they set out.

END OF JUNIOR PROBLEMS SET 2