Junior Division: Problems 1

J1. Six integers have a mean of 32. The median is $33\frac{1}{2}$. The mode is 35, which is not the largest integer. The range is 11. What are the possible sets of 6 integers?

Solution

The mode is 35 and is not the largest number. So 35 occurs twice, and there is one number larger than 35. The median is 33.5 and on one side is 35 so on the other side must be 32.

If the largest number is 36 and the range is 11 then the smallest number must be 25. To make the mean 32 the missing number is 29.

If the largest number is 37 and the range is 11 then the smallest number must be 26. To make the mean 32 the missing number is 27.

If the largest number is 38 and the range is 11 then the smallest number must be 27. To make the mean 32 the missing number would be 25. But this does not work because 27, the smallest number, is more than this.

Also, if the largest number is more than 38 the smallest number will be more than 27. But then the missing number would have to be less than 25. This does not work because the smallest number is more than 27.

So there are two possible sets of integers: 25, 29, 32, 35, 35, 36 and 26, 27, 32, 35, 35, 38.

J2. One of the highlights of the local village social life is the stage production organised by the Amateur Youth Players and the rehearsals are in full swing for *The Gondoliers*. When I called the treasurer the other day he was estimating the costs. The first scene, in case you have forgotten, shows 24 maidens of Venice making up small bunches of red and white roses. He had intended that each girl would have three red and two white roses until he realised that the red roses cost twice as much each as the white ones. He decided to give half the girls three red and two white roses each and the remainder two red and three white roses. He had cut the cost by £3. How much is a red rose?

Solution

Let the cost of a red rose be £R and of white rose be £W.

12 girls had three red roses and two white roses so the cost of these is £$(36R + 24W)$.

The other 12 girls had two red roses and three white roses so the cost of these is £$(24R + 36W)$.

Originally the cost would have been £$(72R + 48W)$ and the new cost is £$(60R + 60W)$.

So the saving is $(72R + 48W) - (60R + 60W) = 12R - 12W = 3$.

But $W = \frac{1}{2}R$ so

$$12R - 12(\tfrac{1}{2}R) = 3$$
$$6R = 3.$$

Hence the cost of a red rose is 50 pence.

J3. At the secondary school down the road Chemistry, English, French, Geography, Maths and Physics are taught by Mr Brown, Mr Jones, Mrs Robinson and Ms Singh.

Each teacher teaches three subjects and each subject is taught by two teachers.

Two of Ms Singh's subjects are also taught by Mr Jones.

Maths is shared by Ms Singh and Mrs Robinson.

Both of the teachers of English also teach French.

Mrs Robinson teaches Chemistry and Mr Jones does not teach Physics.

Who are the two teachers of Geography?

Solution

Maths is shared by Ms Singh and Mrs Robinson. So Mr Brown and Mr Jones do not teach Maths.

Mrs Robinson teaches Chemistry and Mr Jones does not teach Physics.

Both of the teachers of English also teach French. So Mrs Robinson cannot teach English or French as she has space for only one more subject.

	Mr Brown	Mr Jones	Mrs Robinson	Ms Singh
Chemistry			✓	
English			✗	
French			✗	
Geography				
Maths	✗	✗	✓	✓
Physics		✗		

Two of Ms Singh's subjects are also taught by Mr Jones. Ms Singh has only 2 more subjects which are both taught by Mr Jones, so these are either both English and French or both Chemistry and Geography. But if they were Chemistry and Geography there could not be 2 teachers of English and French, so Ms Singh and Mr Jones must both teach English and French.

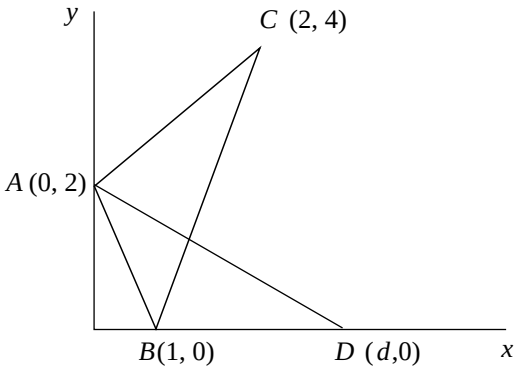
	Mr Brown	Mr Jones	Mrs Robinson	Ms Singh
Chemistry			✓	✗
English	✗	✓	✗	✓
French	✗	✓	✗	✓
Geography				x
Maths	✗	✗	✓	✓
Physics		✗		✗

Physics is therefore taught by Mr Brown and Mrs Robinson. Mrs Robinson has 3 subjects, so does not teach Geography, which must therefore be taught by Mr Brown and Mr Jones. For completeness, Mr Brown's third subject must be Chemistry.

	Mr Brown	Mr Jones	Mrs Robinson	Ms Singh
Chemistry	✓		✓	✗
English	✗	✓	✗	✓
French	✗	✓	✗	✓
Geography	✓	✓	✗	✗
Maths	✗	✗	✓	✓
Physics	✓	✗	✓	✗

Mr Brown and Mr Jones are the two teachers of Geography.

J4.



In the diagram, $D(d, 0)$ lies on the x-axis beyond B. The triangles ABC and ABD have the same area. Determine the value of d.

Solution 1

The triangles both have base AB and the same area. So they must have the same height.

So D must lie on the line through C parallel to AB.

Line AB is $y = -2x + 2$.

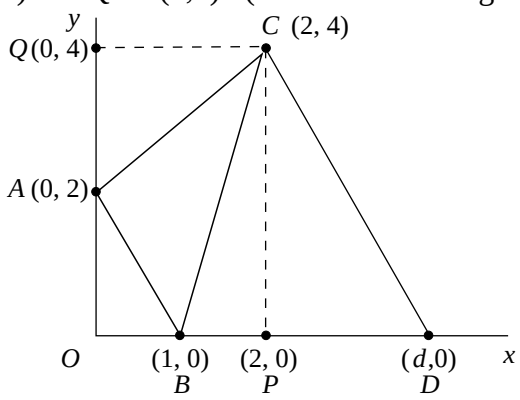
So the parallel line through C is $y = -2x + 8$.

When $y = 0$, $x = 4$.

So $d = 4$.

Solution 2

Let O be the origin (0,0), P be (2, 0) and Q be (0,4). (Shown on the diagram.)



Then

$$\begin{aligned}\text{area } ABC &= \text{area } OPCQ - \text{area } AOB - \text{area } BPC - \text{area } AQC \\ &= 2 \times 4 - \tfrac{1}{2} \times 1 \times 2 - \tfrac{1}{2} \times 1 \times 4 - \tfrac{1}{2} \times 2 \times 2 \\ &= 8 - 1 - 2 - 2 = 3\end{aligned}$$

and

$$\text{area } ABD = \tfrac{1}{2} \times (d - 1) \times 2 = d - 1$$

So $d - 1 = 3$ and therefore $d = 4$.

J5. “Will those in favour of the resolution please hold up their hands?” said the chairperson at a public meeting.

On a count of hands, it appeared that the resolution was carried by a majority of 7. It was then found that, in the excitement of the moment, 6% of those in favour were holding up both their hands. When this had been allowed for, the actual result of the vote proved to be a majority of 2 against the resolution.

How many people voted?

Solution

Let the number of real votes ‘for’ be x.

Then the number of real votes ‘against’ is $x + 2$.

With 6% of those voting for holding up two hands, there were apparently $x \times 1.06$ votes ‘for’.

The apparent majority was then 7, and so

$$x \times 1.06 - (x + 2) = 7$$

$$0.06x = 9$$

$$x = 150$$

So, in all, there were 302 people voting.