Junior Division: Problems 1

J1. Over coffee the other day my friend MacAngus and I were mulling over the passing of time. 'Four years ago', MacAngus said, 'I was four times as old as young Callum and now I'm only three times as old.' 'If you go on like that' I said, 'you'll soon both be the same age.' When we finished laughing over this fallacy, it occurred to me that from what MacAngus said it would be possible to figure out his age. How old is my friend MacAngus?

Solution 1

Let the age now of MacAngus be *m* years and the age now of young Callum be *c* years. Four years ago:

$$m - 4 = 4(c - 4)$$

And now:

m = 3c

So 3c - 4 = 4c - 16 hence c = 12. So MacAngus is 36 years old.

Solution 2

Using	a	table	of	possible	ages.
Using	a	table	01	possible	ages.

Age now		Age 4 years ago		
Callum	MacAngus	Callum	MacAngus	
5	15	1	11	
6	18	2	14	
7	21	3	17	
8	24	4	20	
9	27	5	23	
10	30	6	26	
11	33	7	29	
12	36	8	32	See Note 1
13	39	9	35	See Note 2

Note 1: 4 years ago, MacAngus was 4 times as old as Callum.

Note 2: For older ages, MacAngus is aways less than 4 times as old as Callum.

Answer: So MacAngus is now 36 years old.

J2. Four explorers wish to get one of their number as far as possible into the wilderness from their base. Each explorer can carry supplies for up to 10 days. At any time supplies can be transferred between explorers and individual explorers can return to base, provided they have sufficient supplies for the return journey. Supplies cannot be left unattended in the wilderness.

What is the greatest number of days the lead explorer can travel from the base so that all return safely?

Solution 1

The 4 explorers must set off together, otherwise they will not be able to transfer supplies.

General rule: Each time an explorer returns to base they should retain just enough supplies to reach the base and leave each of the remaining explorers with a full load of supplies.

So the first explorer returns after 2 days, taking 2 days of supplies for themself and topping up the other 3 with 2 days of supplies each, taking their loads back to 10 days supplies.

Then the second explorer returns after 4 days, taking 4 days of supplies for themself and topping up the other 2 with 2 days of supplies each, taking their loads back to 10 days supplies.

The third explorer returns after 6 days, taking 6 days of supplies for themself and topping up the lead explorer with 2 days of supplies, taking their load back to 10 days supplies.

The lead explorer is now 6 days out with 10 days of supplies, so they can travel another 2 days out and still have 8 days supplies to get back to base.

This can be shown in a diagram:



The greatest number of days the lead explorer can travel from base is 8 days.

Note: If the lead explorer trusts their fellow explorers to meet them in the wilderness with more supplies they could get even further!

Solution 2

Each time an explorer returns to base they should have retained just enough supplies to reach the base and leave each of the remaining explorers with a full load of supplies.

Let *x* be the number of days or extra number of days before each man leaves to return.

First man uses 2x supplies.

Second man uses 4x supplies.

Third man uses 6x supplies.

Last man uses 8x supplies.

So

$$8x + 6x + 4x + 2x = 40$$
$$20x = 40$$
$$x = 2$$

The greatest number of days the lead explorer can travel from base is 8 days.

So the first explorer must return after 2 days, taking 2 days of supplies and topping up the others with 2 days of supplies each.

J3. An integer n, between 100 and 999 inclusive, is chosen at random. What is the probability that the sum of the digits of n is 23?

Solution

There are 899 + 1 = 900 numbers to choose from.

The possible sets of 3 digits with sum 23 are

Where a digit is repeated there are 3 possible positions for the different digit and so 3 possible 3 digit numbers.

Where there is no repetition there are 3 ways of choosing the first digit, and 2 ways of choosing the second digit, giving $2 \times 3 = 6$ ways of forming a 3 digit number.

So there are 3 + 6 + 3 + 3 = 15 possible numbers where the sum of the digits is 23.

So the probability of choosing a number where the sum of the digits is 23 is

$$\frac{15}{900} = \frac{1}{60}$$

J4. A van delivering maths textbooks travels to a school at an average speed of 60mph. Due to a mechanical problem the van could only do the return journey at an average speed of 40mph. What was the van's average speed over the entire journey?

Solution

Let the distance be *d* miles and the times (in hours) be T_1 and T_2 .

$$T_1 = \frac{d}{60}$$
$$T_2 = \frac{d}{40}$$

The total distance is 2d and the total time is

$$T_1 + T_2 = \frac{d}{60} + \frac{d}{40}$$
$$= \frac{2d + 3d}{120} = \frac{5d}{120} = \frac{d}{24}.$$

So the average speed is

$$\frac{2d}{T_1 + T_2} = \frac{2d}{\frac{d}{24}} = \frac{48d}{d} = 48 \text{ mph}.$$

 $\begin{array}{c} R \\ R \\ A \\ B \\ P \\ Q \\ C \\ D \end{array}$

In the diagram, points *B*, *P*, *Q* and *C* lie on line segment *AD*. The semi-circle with diameter *AC* has centre *P* and the semi-circle with diameter *BD* has centre *Q*. The two semi-circles intersect at *R*. If $\angle PRQ = 30$ degrees, determine the size of $\angle ARD$.

Solution



Let $\angle RAD = a$ and $\angle RDA = d$, both measured in degrees.

Then $\angle ARP = \angle RAP = a$ since both are base angles of isosceles triangle *ARP*, where *AP* and *RP* are both radii of the circle with centre *P*.

Also $\angle DRQ = \angle RDQ = d$ since both are base angles of isosceles triangle *DRQ*, where *DQ* and *RQ* are both radii of the circle with centre *Q*.

The sum of the angles of the triangle ARD is 180 degrees, so

$$2a + 30 + 2d = 180$$

$$a + d = 75$$

$$\angle ARD = \angle ARP + \angle PRQ + \angle DRQ = a + 30 + d = 30 + 75 = 105$$

$$\angle ARD = 105 \text{ degrees}.$$

J5.