

# The Scottish Mathematical Council

www.scot-maths.co.uk

## MATHEMATICAL CHALLENGE 2023–2024

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

*CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE*

*The Edinburgh Mathematical Society, The Maxwell Foundation,*

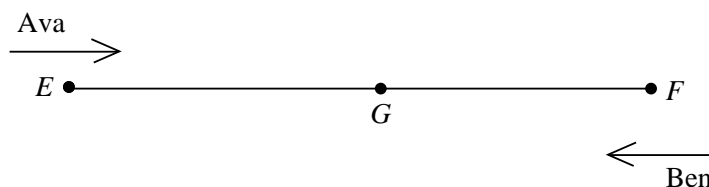
*The London Mathematical Society and The Scottish International Education Trust.*

The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

Particular thanks are due to the Universities of Aberdeen, Edinburgh, Glasgow, Heriot Watt, St Andrews, Stirling, Strathclyde and to George Heriot's School, Gryffe High School and Kelvinside Academy.

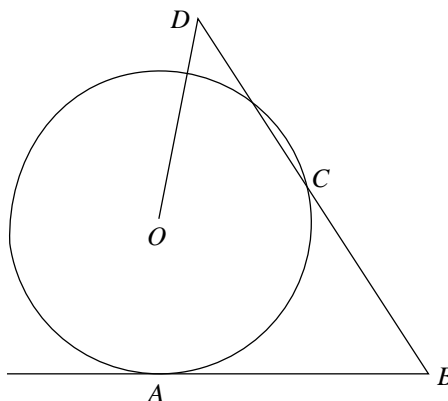
### Senior Division: Problems 2

- S1.** Ava drove from town  $E$  to town  $F$  at a constant speed of 60 mph. Ben drove from  $F$  to  $E$  along the same road also at a constant speed. They started their journeys at the same time and passed each other at point  $G$ .



Ava drove from  $G$  to  $F$  in 16 minutes. Ben drove from  $G$  to  $E$  in 25 minutes. Determine Ben's constant speed.

- S2.** The numbers  $p, q, r, s$  and  $t$  are consecutive positive integers arranged in increasing order.  $p + q + r + s + t$  is a perfect cube and  $q + r + s$  is a perfect square. Find the smallest possible value of  $r$ .
- S3.** If  $f(x) = x - 3$  and  $g(f(x)) = x^2 - 10$ , determine an expression for  $g(x)$ .
- S4.** In the diagram,  $AB$  is tangent to the circle with centre  $O$  and radius  $r$ . The length of  $AB$  is  $p$ . Point  $C$  is on the circle and  $D$  is outside the circle so that  $BCD$  is a straight line, as shown. Also  $BC = CD = DO = q$ .



Prove that  $p^2 = q^2 + r^2$ .

**SEE OVER FOR QUESTION S5.**



# Mathematical Challenge Problems 2

SENIOR DIVISION 2023-2024

PLEASE USE CAPITALS TO COMPLETE

SURNAME

OTHER NAME(S)  
(underline the one  
you prefer)

SCHOOL

AGE

YEAR OF STUDY

FOR OFFICIAL USE

Marker

Marks

1	2	3	4	5

Total

— — — — - CUT ALONG HERE — — — —

Please write your solutions on A4 paper and staple the above form to them.

PLEASE WRITE YOUR NAME ON EVERY PAGE.

Send your entry through your school to the section organiser.

For further information on the competition, please see the School Materials which have been distributed to schools. A copy of these Materials can be obtained from

<http://www.wpr3.co.uk/MC/materials/index.html>

There are separate links for primary and secondary schools. This page also includes a list of authorities in each section and names and addresses of section organisers.

- S5.** I have two bags containing coloured balls. The first bag contains five balls, all of which are red, whilst all the balls in the second bag are blue. I transfer one of the balls in the first bag to the second, then pick at random a ball from the second bag and transfer it to the first bag. I now pick a ball at random from the first bag and transfer it to the second bag. If the probability of a ball picked at random from the second bag being blue is then  $\frac{3}{5}$ , how many blue balls were there in the second bag originally?

**END OF PROBLEM SET 2**

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

23 February 2024

For information about Mathematical Challenge, look on the SMC website:

[www.scot-maths.co.uk](http://www.scot-maths.co.uk)

For more practice on past papers visit:

[www.wpr3.co.uk/MC-archive/](http://www.wpr3.co.uk/MC-archive/)