MATHEMATICAL CHALLENGE 2019–2020

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel,
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Senior Division: Problems 2

S1. A positive integer ends in the digit 4 and has the property that it becomes four times as large when the 4 is moved from the end and placed at the front. What is the smallest such number?

S2.

Let $ABCD$ be a quadrilateral. Let $A'$ be the midpoint of $AB$, $B'$ the mid-point of $BC$, $C'$ the mid-point of $CD$ and $D'$ the mid-point of $AD$. Draw the lines $A'C'$ and $B'D'$ and let $a$, $b$, $c$, $d$ be the areas of the four minor quadrilaterals as shown in the figure. Prove that $a + c = b + d$.

S3. Find all values of $x$ such that

$$\log_2 (3x + 2) + \log_2 (4x - 4) = 3.$$ 

S4.

The diagram shows three circles, $C_1$, $C_2$ and $C_3$ and two parallel lines, $L_1$ and $L_2$. $C_1$ touches both the lines, $C_2$ touches $L_1$ and $C_1$, and $C_3$ touches $C_1$, $C_2$ and $L_2$. The radius of $C_2$ is 16 and the radius of $C_3$ is 9. Find the radius of $C_1$.

SEE OVER FOR QUESTION S5.
### S5. If $p$ and $q$ are positive integers, $\max(p, q)$ is the maximum of $p$ and $q$ and $\min(p, q)$ is the minimum of $p$ and $q$. So for example $\max(3, 6) = 6$ and $\min(3, 6) = 3$.

Determine the number of ordered pairs $(x, y)$ which satisfy the equation

$$\max(70, \min(x, y)) = \min(\max(70, x), y)$$

where $x$ and $y$ are positive integers with $x \leq 100$ and $y \leq 100$.

**END OF PROBLEM SET 2**

**CLOSING DATE FOR RECEIPT OF SOLUTIONS**: 21 February 2020

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