



The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2021–2022

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation,

The London Mathematical Society and The Scottish International Education Trust.

The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

Particular thanks are due to the Universities of Aberdeen, Edinburgh, Glasgow, Heriot Watt, St Andrews, Stirling, Strathclyde and to George Heriot's School, Gryffe High School and Kelvinside Academy.

Senior Division: Problems 2

S1. Three thin metal rods of lengths 9, 12 and 15 are welded together to form a right-angled triangle, which is held in a horizontal position. A solid sphere of radius 5 rests in the triangle so that it touches each of the three sides. Assuming that the thickness of the rods can be neglected, how high above the plane of the triangle is the top of the sphere?

S2. Determine all solutions of the equation

$$(x - y)^2 + x^2 = 25$$

where x and y are integers and $x \geq 0$.

S3. Lisa is tiling her long and narrow front hall. The hall is 2 tiles wide and 12 tiles long. She is going to use exactly 11 black tiles and exactly 13 white tiles. Determine the number of distinct ways of tiling the hall so that no two black tiles are adjacent (that is, no two black tiles have a common edge).

S4. The circle $(x - p)^2 + y^2 = r^2$ has centre C and the circle $x^2 + (y - p)^2 = r^2$ has centre D . The circles intersect at two distinct points A and B , with x coordinates a and b respectively.

(a) Prove that $a + b = p$ and $a^2 + b^2 = r^2$.

(b) If p and r are integers find the minimum possible distance between A and B . Find positive integers p and r , each greater than 1, that give this distance.

S5. Determine all values of x for which

$$(\sqrt{x})^{\log_{10} x} = 100.$$

END OF PROBLEM SET 2

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

11 February 2022

For information about Mathematical Challenge, look on the SMC web site:

www.scot-maths.co.uk



SMC

Mathematical Challenge Problems 2

SENIOR DIVISION 2021-2022

PLEASE USE CAPITALS TO COMPLETE

SURNAME

OTHER NAME(S)
(underline the one
you prefer)

SCHOOL

AGE

YEAR OF STUDY

FOR OFFICIAL USE

Marker

Marks

1	2	3	4	5

Total

— — — — - **CUT ALONG HERE** — — — —

Please write your solutions on A4 paper and staple the above form to them.

PLEASE WRITE YOUR NAME ON EVERY PAGE.

Send your entry through your school to the section organiser.

For further information on the competition, please see the School Materials which have been distributed to schools. A copy of these Materials can be obtained from

<http://www.wpr3.co.uk/MC/materials/index.html>

There are separate links for primary and secondary schools. This page also includes a list of authorities in each section and names and addresses of section organisers.
