The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2023−2024

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation,

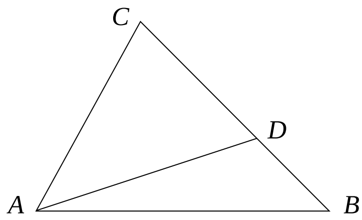
The London Mathematical Society and The Scottish International Education Trust.

The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

Particular thanks are due to the Universities of Aberdeen, Edinburgh Napier, Moray House, St Andrews, Stirling, Strathclyde and to George Heriot's School, Gryffe High School and Kelvinside Academy.

Senior Division: Problems 1

S1.



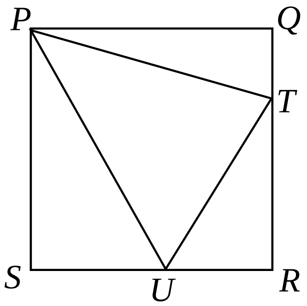
In the diagram, $2\angle BAC = 3\angle ABC$ and D lies on BC such that $\angle DAC = 2\angle DAB$. Suppose that $BC = a$, $AC = b$, $AB = c$, $AD = d$ and $CD = e$.

Find expressions for d and e in terms of a, b and c only.

S2. A school assembly hall has a rectangular array of chairs. There are exactly 12 boys seated in each of the r rows and exactly 10 girls seated in each of the c columns. There are fewer than 1000 boys and girls in the school. There is just one empty chair.

How many chairs are there in the assembly hall?

S3.



In the diagram, $PQRS$ is a square with sides of length 2. Points T and U are on sides QR and RS respectively such that $\angle TPU = 45°$.

Determine the minimum possible perimeter of triangle RTU.

S4. George throws three unbiased dice and removes all of the dice that come up with a 5 or 6. Martha then throws the dice that remain, if any. Determine the probability that exactly one of Martha's dice shows a 5 or 6.

SEE OVER FOR QUESTION S5.

Mathematical Challenge Problems 1

SMC

SENIOR DIVISION 2023-2024

PLEASE USE CAPITALS TO COMPLETE

SURNAME			
OTHER NAME(S) (underline the one you prefer)			
SCHOOL			
AGE		YEAR OF STUDY	S

FOR OFFICIAL USE

Marker

Marks

1	2	3	4	5

Total	

Please write your solutions on A4 paper and staple the above form to them.
PLEASE WRITE YOUR NAME ON EVERY PAGE.
Send your entry through your school to the section organiser.

For further information on the competition, please see the School Materials which have been distributed to schools. A copy of these Materials can be obtained from

http://www.wpr3.co.uk/MC/materials/index.html

There are separate links for primary and secondary schools. This page also includes a list of authorities in each section and names and addresses of section organisers.

S5. The irrational number $\sqrt{2}$ can be written as a series of continued fractions in the following way

$$\begin{aligned}\sqrt{2} &= 1 + \left(\sqrt{2} - 1\right) = 1 + \frac{1}{\sqrt{2}+1} \\ &= 1 + \frac{1}{2 + \left(\sqrt{2}-1\right)} = 1 + \frac{1}{2 + \frac{1}{\sqrt{2}+1}} \\ &= 1 + \frac{1}{2 + \frac{1}{2+\left(\sqrt{2}-1\right)}} = 1 + \frac{1}{2 + \frac{1}{2+\frac{1}{\sqrt{2}+1}}}\end{aligned}$$

This process can be continued. If we stop after n steps and ignore the term containing $\sqrt{2}$ we get a rational number $\frac{p_n}{q_n}$. So

$$\frac{p_1}{q_1} = 1, \qquad \frac{p_2}{q_2} = 1 + \frac{1}{2} = \frac{3}{2}, \qquad \frac{p_3}{q_3} = 1 + \frac{1}{2+\frac{1}{2}} = \frac{7}{5}$$

and so on.

Show that, for all odd integers n, $\frac{p_n}{q_n} < \sqrt{2}$ and for all even n, $\frac{p_n}{q_n} > \sqrt{2}$.

END OF PROBLEM SET 1

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

3 November 2023

Look out for Problems 2 in late November!

For information about Mathematical Challenge, look on the SMC web site:
www.scot-maths.co.uk