The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2021−2022

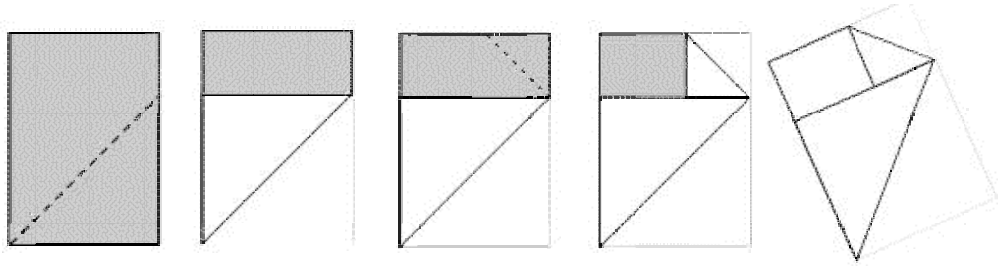
Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, The Maxwell Foundation,
The London Mathematical Society and The Scottish International Education Trust.
The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.
Particular thanks are due to the Universities of Aberdeen, Edinburgh, Glasgow, St Andrews, Stirling, Strathclyde and to George Heriot's School, Gryffe High School and Kelvinside Academy.

Senior Division: Problems 1

S1. A piece of A4 paper remains the same shape when it is folded in half i.e. the ratio of length to breadth remains the same. Show that this ratio is $\sqrt{2}$: 1.

Take a piece of A4 paper and fold as shown below to create a shape.



Prove that this shape is a kite and find its area.

S2. The sum of the cubes of three consecutive integers is 33 times the middle integer. Find the three integers.

S3. If $f(x) = px + q$ and $f(f(f(x))) = 8x + 21$, and if p and q are real numbers, find the value of $p + q$.

S4.

1	7	31
2	8	32
3	9	33

In each column of this grid the first number is the smallest number that cannot be achieved by adding together some combination of the numbers in the preceding columns. What will the numbers be in the 4th column? What will the numbers be in the nth column? Explain.

SEE OVER FOR QUESTION S5.

SMC

Mathematical Challenge Problems 1

SENIOR DIVISION 2021-2022

PLEASE USE CAPITALS TO COMPLETE

SURNAME	
OTHER NAME(S) (underline the one you prefer)	
SCHOOL	
AGE	
YEAR OF STUDY	S

FOR OFFICIAL USE

Marker

Marks

1	2	3	4	5

Total	

– – – – - CUT ALONG HERE – – – –

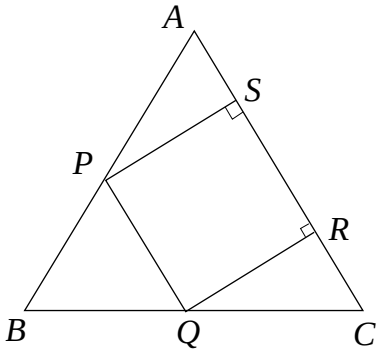
Please write your solutions on A4 paper and staple the above form to them.
PLEASE WRITE YOUR NAME ON EVERY PAGE.
Send your entry through your school to the section organiser.

For further information on the competition, please see the School Materials which have been distributed to schools. A copy of these Materials can be obtained from

http://www.wpr3.co.uk/MC/materials/index.html

There are separate links for primary and secondary schools. This page also includes a list of authorities in each section and names and addresses of section organisers.

S5. An equilateral triangle *ABC* has side length 2. A square, *PQRS*, is such that *P* lies on *AB*, *Q* lies on *BC*, and *R* and *S* lie on *AC* as shown. The points *P*, *Q*, *R* and *S* move so that *P*, *Q* and *R* always remain on the sides of the triangle and *S* moves from *AC* to *AB* through the interior of the triangle. If the points *P*, *Q*, *R* and *S* always form the vertices of a square, show that the path traced out by *S* is a straight line parallel to *BC*.



END OF PROBLEM SET 1

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

5 November 2021

Look out for Problems 2 in late November!

For information about Mathematical Challenge, look on the SMC web site:

www.scot-maths.co.uk