The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2021−2022

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, The Maxwell Foundation,
The London Mathematical Society and The Scottish International Education Trust.
The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.
Particular thanks are due to the Universities of Aberdeen, Edinburgh, Glasgow, Heriot Watt, St Andrews, Stirling, Strathclyde and to George Heriot's School, Gryffe High School and Kelvinside Academy.

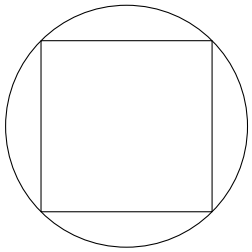
Middle Division: Problems 2

M1. Three expert logicians played a game with a set of 11 cards each with a different two-digit prime number below 50. Each drew a card and held it up so that they could only see the number on their own card but could not see the other numbers. Ali, Bobby and Charlie in turn were then asked two questions, namely “Is your number the smallest of the three?” and “Is your number the largest of the three?”. In the first round all three answered “Don't know” to both questions. The same happened in rounds two and three. In round 4 Ali answered “No” to the first question.

What numbers did each logician have?

M2. Alistair and Jonny cross a lake by swimming and using a one-seat canoe. Each swims at 2 km/hour and paddles the canoe at 7 km/hour. They set off from the same point at the same time, heading straight for the boathouse at the opposite side, with Alistair swimming and Jonny paddling the canoe. After a while Jonny stops paddling, gets out of the canoe and immediately starts swimming. When Alistair reaches the canoe, which has not moved since Jonny started swimming, Alistair climbs in and immediately starts paddling. After 90 minutes they both arrive at the boathouse together. For how long was the canoe stationary?

M3. During one of the frequent staffroom discussions on gardening the Maths teacher handed the Chemistry teacher this diagram:



The diagram showed a flowerbed in the middle of the Maths teacher's lawn. Originally it was just a square but she decided to cut out a bit more turf to make it circular so that it could hold more roses. Interestingly, when it was square the perimeter in metres was the same number as the area in square metres. The Chemistry teacher asked how many more square metres of flowerbed she had now than she had previously but before she could answer the Physics teacher chimed in with “I know”.

Can you figure it out?

M4. Three thin metal rods of lengths 9, 12 and 15 are welded together to form a right-angled triangle, which is held in a horizontal position. A solid sphere of radius 5 rests in the triangle so that it touches each of the three sides. Assuming that the thickness of the rods can be neglected, how high above the plane of the triangle is the top of the sphere?

SEE OVER FOR QUESTION M5.

SMC

Mathematical Challenge Problems 2

MIDDLE DIVISION 2021-2022

PLEASE USE CAPITALS TO COMPLETE

SURNAME	
OTHER NAME(S) (underline the one you prefer)	
SCHOOL	
AGE	YEAR OF STUDY S

FOR OFFICIAL USE

Marker

Marks

1	2	3	4	5

Total	

– – – – CUT ALONG HERE – – – –

Please write your solutions on A4 paper and staple the above form to them.
PLEASE WRITE YOUR NAME ON EVERY PAGE.
Send your entry through your school to the section organiser.

For further information on the competition, please see the School Materials which have been distributed to schools. A copy of these Materials can be obtained from

http://www.wpr3.co.uk/MC/materials/index.html

There are separate links for primary and secondary schools. This page also includes a list of authorities in each section and names and addresses of section organisers.

M5. Determine all solutions of the equation

$$(x - y)^2 + x^2 = 25$$

where x and y are integers and $x \geqslant 0$.

END OF PROBLEM SET 2

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

11 February 2022

For information about Mathematical Challenge, look on the SMC web site:
www.scot-maths.co.uk