Secondary Mathematical Challenges

Welcome to the second round of the 2023-2024 Scottish Secondary Mathematical Challenges.
This package contains

This Welcome Page (**including Section Information**)

Round 2 Questions

In 2023-2024, the name of the Section Organiser is not on the question paper. Their details are on the website but are repeated here for convenience.

Please take great care to use the correct one.

Section 1

Aberdeen City; Aberdeenshire; Highland; Moray; Orkney Islands; Shetland Islands; Western Isles

Dr Richard Hepworth (r.hepworth@abdn.ac.uk)
Mathematical Challenge
Department of Mathematical Sciences, University of Aberdeen,
Aberdeen AB24 3UE

Section 2

Angus; Clackmannanshire; Dundee City; Falkirk; Fife; Perth & Kinross; Stirling

Dr Jean Reinaud (jnr1@st-andrews.ac.uk)
Mathematical Institute, University of St Andrews,
St Andrews, Fife KY16 9SS

Section 3

East Lothian; Edinburgh City; Midlothian; Scottish Borders; West Lothian

Andrew Gallacher (A.Gallacher@napier.ac.uk)
Head of Teacher Education, Edinburgh Napier University, School of Applied Sciences,
Room 2.B.37, Sighthill Court, Edinburgh EH11 4BN

Section 4

Argyll & Bute; Dumfries & Galloway; East Ayrshire; East Dunbartonshire;
East Renfrewshire; Glasgow City; Inverclyde; North Ayrshire; North Lanarkshire;
Renfrewshire; South Ayrshire; South Lanarkshire; West Dunbartonshire

Scottish Mathematical Challenge Organiser (wpr3145@gmail.com),
Department of Mathematics and Statistics, University of Strathclyde,
26 Richmond Street, Glasgow G1 1XH

The competition timetable for 2023-2024 is as follows:

Set No.	***Last date for receipt of questions by schools***	***Last date for receipt of solutions from pupils***
I	Friday 25 August 2023	Friday 3 November 2023
II	Friday 24 November 2023	Friday 23 February 2024

If there are organisational difficulties you may contact me, Bill Richardson, (wpr3145@gmail.com).

Books of past questions are still available but it seems unlikely that any more will be printed so questions and solutions for 2006-2018 can be accessed at:

www.wpr3.co.uk/MC-archive/

The Scottish Mathematical Council

MC homepage: www.scot-maths.co.uk/

MATHEMATICAL CHALLENGE 2023−2024

A national problem solving competition for schools in Scotland

S E C O N D A R Y D I V I S I O N S

GUIDELINES FOR TEACHERS

1. ***Mathematical Challenge*** is a problem-solving competition which goes back to 1976. The Challenge is open to all students educated in Scotland. Its aim is to promote mathematics as a source of interest and pleasurable achievement through challenging problems which require only elementary techniques and simple logic.

   Please ensure that all teachers involved in the competition see these Guidelines.

How Mathematical Challenge operates

2. There are four divisions: JUNIOR for Sl and S2, MIDDLE for S3 and S4, SENIOR for S5 and S6, and PRIMARY (for which a separate circular is available).

   Pupils may enter only one division and must specify that division on their first entry.
   Please contact your local organiser, whose name and address are on the proforma on page 2 of each problem sheet and in the Contacts section of the Web pages, if there is any doubt about divisions, or if further information is required.

3. There are no written examinations. For the Junior, Middle and Senior Divisions, two sets of five problems each will be available for schools to download according to a timetable outlined in §13 below. Problems for different divisions will be on separate sheets. Some problems may be common to different divisions. The problems will also be available from the Mathematical Challenge Web pages (see above for address).

4. **A registration fee is required from participating schools. For a secondary school the fee is £20 for the first 10 entrants and half this amount for each subsequent batch of 10 entrants or part thereof. A fee form is included with the downloadable pack of materials.**
   For individual participants NOT entering through a school, the fee is £8.

Entries and Marking

5. **Entries must be the unaided efforts of individual pupils. Group working is not appropriate in Mathematical Challenge.** Participants may consult books or the internet for information on facts or on how to tackle problems. Whilst teachers or parents may give guidance on interpretation of wording, **they should not be involved in the solution of a problem.** Furthermore, **the work should not interfere with normal teaching and in no circumstances should it be a class assignment.**

6. All Sections must use the software package to assist in the processing of the results. A **Record of Entries must be made electronically by the school**, or it will not be possible to process the results.

   - Go to the marks website: https://www.scottishmathschallenge.org.uk/
   - Choose "School Login" and enter your login details or "Register here" to set up a new account.
   - When you have logged in, go to "Add/Edit Entrants" _ enter the names and school year of each entrant.
   - The marks will eventually appear on the "Marks page".
   - Messages from the organiser may also appear there on the first page from time to time.

   Use a paper copy of the 'Printable version of details and entrants' from the marks website as a cover sheet for the school's entries. This contains the school details and the alphabetical list of entrants in each section, as entered on the website. All entries submitted will be marked even if earlier problem sets are missed.

7. Entries will not be returned. Entrants should keep a copy of their solutions. The Scottish Mathematical Council reserves the right to publish good solutions in its Journal.

8. **Participants should explain their solutions as fully as they can**. Marks will be given for explanations of answers not just for the answers themselves. **We should be most grateful if teachers would stress this point**. Incomplete or incorrect answers may gain some credit.

   In outline, the marking scheme for each problem is as follows:

   4 : a completely correct solution, with full explanation.

   3 : a solution, with explanation, which is correct apart from a minor slip or omission of a special case.

   2 : a solution with explanation which contains a serious error or omission, but which nevertheless involves good ideas.

   1 : there is an indication of an interesting idea or method, but not necessarily one which could lead to a correct solution.

   A *bonus* mark may be given for a completely correct solution, with full explanation, which contains additional good ideas, such as a successful generalisation of the problem.

   A solution in which an answer is given without any explanation will normally be awarded no marks, even if the answer is correct. However, correct working may be accepted as providing an explanation, so long as the various steps are clear.

9. No problems set in ***Mathematical Challenge*** require the use of a computer package (e.g. a spreadsheet) to obtain a correct solution. If computer software is used, then a proper mathematical explanation of its use is essential.

Awards

10. There are three classes of award: ***Gold***, ***Silver*** and ***Bronze***. Award winners will be selected primarily on the basis of the total number of marks obtained over both sets of Problems. Special circumstances for individual entrants may be taken into account.

11. All award winners will qualify for certificates. Where an award ceremony can be arranged, the most successful entrants will be invited to attend to receive their certificates and Mathematical Challenge mugs. Certificates not presented at a ceremony will be sent by post.

Important notes

12. Large numbers of entries can impose a considerable strain on markers and on organisers. Local organisers may have to set limits on the total numbers of entries per school. Schools submitting large numbers of entries may be asked to provide additional markers. Any such markers would not mark entries from their own schools.

13. The timetable for 2022-2023 is as follows:

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14. **The problems of earlier sessions form an excellent resource.** Those for the years 1991-92 to 2005-2006, including solutions, are available in the books *Mathematical Challenges III*, *Mathematical Challenges IV*, *Mathematical Challenges V* and *Mathematical Challenges VI* which are published by The Scottish Mathematical Council. Copies can be obtained from Bill Richardson, Kintail, Longmorn, Elgin IV30 8RJ, prices £7.50, £8, £8, £8 respectively.

    In addition, it seems unlikely that any further books will be printed so questions and solutions for 2006-2021 can be accessed at: **www.wpr3.co.uk/MC-archive/**

    Comments on the usefulness of these to wpr3145@gmail.com would be welcome.

15. For other information, please contact your local organiser, whose name and address are given in the Contacts section of the Mathematical Challenge Web pages

    www.scot-maths.co.uk

    as well as on the materials download menu page

    www.wpr3.co.uk/MC/materials

The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2023−2024

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, The Maxwell Foundation,
The London Mathematical Society and The Scottish International Education Trust.
The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.
Particular thanks are due to the Universities of Aberdeen, Edinburgh, Glasgow, Heriot Watt, St Andrews, Stirling, Strathclyde and to George Heriot's School, Gryffe High School and Kelvinside Academy.

Junior Division: Problems 2

J1. A cruise liner, which is 100m long, is sailing at 20 km/hr. As its bow passes a buoy, a passenger starts to walk from bow to stern at 4 km/hr. When he reaches the stern, how far past the buoy will he be?

J2. The directors of a company which specialises in the construction of cubes are planning to build a car park at the front of their building. This car park is in the shape of a rectangle, with a total area of 3055 square metres. They make a request to the builders that the car park is made up of square slabs, all of different sizes, and have calculated that it can be done using squares of side 3, 5, 6, 11, 17, 19, 22, 23, 24 and 25 metres.

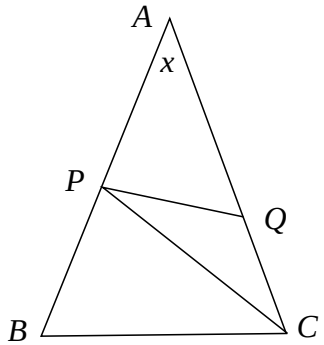
What must be the dimensions of the car park? How can the slabs be placed to fit?

J3. At a school, 15 students were absent on Monday, 12 were absent on Tuesday and 9 were absent on Wednesday. If none of the students was absent on all three days, what is the smallest possible total number of students that were absent on at least one day?

Justify your answer.

J4. Triangle ABC has $AB = AC$ and $\angle BAC = x$ is less than 60°. Point P lies on AB such that $CB = CP$. Point Q lies on AC such that $CQ = PQ$.

Determine $\angle CQP$ in terms of x.



J5. One hundred years ago there was a gathering to present an award to a local teacher in recognition of many years of service.

The women there numbered four-fifths of the men, 40% of whom were unmarried. Half of the married women were accompanied by their husbands and a quarter of the married men by their wives. Thirty of the women were unmarried.

How many people were there at the gathering?

END OF PROBLEM SET 2

CLOSING DATE FOR RECEIPT OF SOLUTIONS: **23 February 2024**

For more practice on past papers visit: www.wpr3.co.uk/MC-archive/

SMC

Mathematical Challenge Problems 2

JUNIOR DIVISION 2023-2024

PLEASE USE CAPITALS TO COMPLETE

SURNAME

OTHER NAME(S) (underline the one you prefer)

SCHOOL

AGE

YEAR OF STUDY S

FOR OFFICIAL USE

Marker

Marks

1	2	3	4	5

Total

– – – – - CUT ALONG HERE – – – –

Please write your solutions on A4 paper and staple the above form to them.

PLEASE WRITE YOUR NAME ON EVERY PAGE.

Send your entry through your school to the section organiser.

For further information on the competition, please see the School Materials which have been distributed to schools. A copy of these Materials can be obtained from

http://www.wpr3.co.uk/MC/materials/index.html

There are separate links for primary and secondary schools. This page also includes a list of authorities in each section and names and addresses of section organisers.

The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2023−2024

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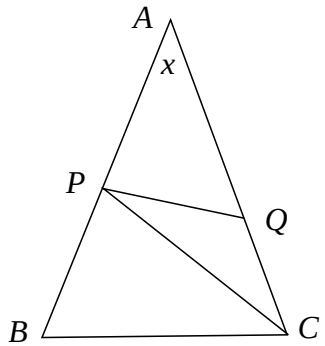
Middle Division: Problems 2

M1. Triangle ABC has $AB = AC$ and $\angle BAC = x$ is less than 60°. Point P lies on AB such that $CB = CP$. Point Q lies on AC such that $CQ = PQ$.

Determine $\angle CQP$ in terms of x.



M2. One hundred years ago there was a gathering to present an award to a local teacher in recognition of many years of service.

The women there numbered four-fifths of the men, 40% of whom were unmarried. Half of the married women were accompanied by their husbands and a quarter of the married men by their wives. Thirty of the women were unmarried.

How many people were there at the gathering?

M3. A market trader sells fruit. As a special offer, he has made up baskets of fruit. The first has four bananas, three oranges and two apples and costs £2.90; the second has three bananas, two oranges and four apples and costs £2.60; the third has two bananas, four oranges and three apples and costs £2.60. Individual fruit bought costs 20% more than the corresponding price in any of the special offer baskets. I don't fancy any of the mixtures in the baskets, but I do want at least two bananas, two oranges and three apples. So I buy bananas, oranges and apples individually and pay £3.12. How many of each fruit do I get?

SEE OVER FOR QUESTIONS M4 & M5.

SMC

Mathematical Challenge Problems 2

MIDDLE DIVISION 2023-2024

PLEASE USE CAPITALS TO COMPLETE

SURNAME	
OTHER NAME(S) (underline the one you prefer)	
SCHOOL	
AGE	
YEAR OF STUDY	S

FOR OFFICIAL USE

Marker

Marks

1	2	3	4	5

Total

– – – – - CUT ALONG HERE – – – –

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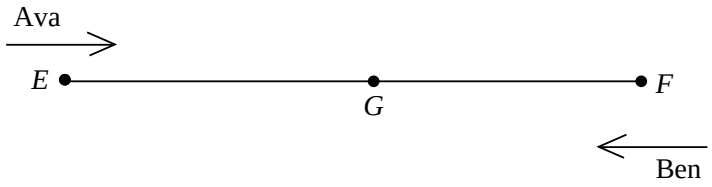
M4. Ava drove from town *E* to town *F* at a constant speed of 60 mph. Ben drove from *F* to *E* along the same road also at a constant speed. They started their journeys at the same time and passed each other at point *G*.



Ava drove from *G* to *F* in 16 minutes. Ben drove from *G* to *E* in 25 minutes. Determine Ben's constant speed.

M5. The numbers p, q, r, s and t are consecutive positive integers arranged in increasing order. $p + q + r + s + t$ is a perfect cube and $q + r + s$ is a perfect square. Find the smallest possible value of r.

END OF PROBLEM SET 2

CLOSING DATE FOR RECEIPT OF SOLUTIONS:

23 February 2024

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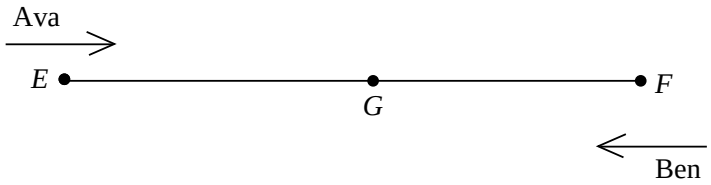
Senior Division: Problems 2

S1. Ava drove from town E to town F at a constant speed of 60 mph. Ben drove from F to E along the same road also at a constant speed. They started their journeys at the same time and passed each other at point G.

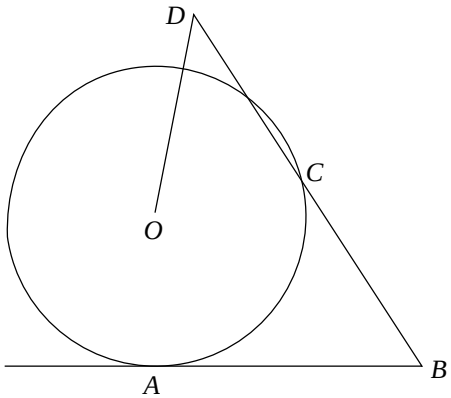


Ava drove from G to F in 16 minutes. Ben drove from G to E in 25 minutes. Determine Ben's constant speed.

S2. The numbers p, q, r, s and t are consecutive positive integers arranged in increasing order. $p + q + r + s + t$ is a perfect cube and $q + r + s$ is a perfect square. Find the smallest possible value of r.

S3. If $f(x) = x - 3$ and $g(f(x)) = x^2 - 10$, determine an expression for $g(x)$.

S4. In the diagram, AB is tangent to the circle with centre O and radius r. The length of AB is p. Point C is on the circle and D is outside the circle so that BCD is a straight line, as shown. Also $BC = CD = DO = q$.



Prove that $p^2 = q^2 + r^2$.

SEE OVER FOR QUESTION S5.

SMC

Mathematical Challenge Problems 2

SENIOR DIVISION 2023-2024

PLEASE USE CAPITALS TO COMPLETE

SURNAME	
OTHER NAME(S) (underline the one you prefer)	
SCHOOL	
AGE	YEAR OF STUDY S

FOR OFFICIAL USE

Marker

Marks

1	2	3	4	5

Total

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S5. I have two bags containing coloured balls. The first bag contains five balls, all of which are red, whilst all the balls in the second bag are blue. I transfer one of the balls in the first bag to the second, then pick at random a ball from the second bag and transfer it to the first bag. I now pick a ball at random from the first bag and transfer it to the second bag. If the probability of a ball picked at random from the second bag being blue is then $\frac{3}{5}$, how many blue balls were there in the second bag originally?

END OF PROBLEM SET 2

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