

## Secondary Mathematical Challenges

Welcome to the second round of the 2018-2019 Scottish Secondary Mathematical Challenges.

This package contains

This Welcome Page (**including Section Information**)

Round 2 Questions

In 2018-2019, the name of the Section Organiser is not on the question paper. Their details are on the website but are repeated here for convenience.

**Please take great care to use the correct one.**

### Section 1

Aberdeen City; Aberdeenshire; Highland; Moray; Orkney Islands; Shetland Islands;  
Western Isles

Dr William Turner (w.turner@abdn.ac.uk)

Mathematical Challenge

Department of Mathematical Sciences, University of Aberdeen,  
Aberdeen AB24 3UE

### Section 2

Angus; Clackmannanshire; Dundee City; Falkirk; Fife; Perth & Kinross; Stirling

Dr Jean Reinaud (jnr1@st-andrews.ac.uk)

Mathematical Challenge

Mathematical Institute, University of St Andrews,  
St Andrews, Fife KY16 9SS

### Section 3

East Lothian; Edinburgh City; Midlothian; Scottish Borders; West Lothian

Dr Lotte Hollands (l.hollands@hw.ac.uk)

Mathematical Challenge

Department of Mathematics,  
Heriot Watt University, Edinburgh EH14 4AS

### Section 4

Argyll & Bute; Dumfries & Galloway; East Ayrshire; East Dunbartonshire;  
East Renfrewshire; Glasgow City; Inverclyde; North Ayrshire; North Lanarkshire;  
Renfrewshire; South Ayrshire; South Lanarkshire; West Dunbartonshire

Dr Chris Athorne (Christopher.Athorne@maths.gla.ac.uk)

Department of Mathematics, University of Glasgow,  
University Gardens Glasgow G12 8QW

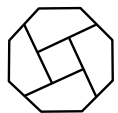
The competition timetable for 2018-2019 is as follows:

<i>Set</i>	<i>Last date for receipt of questions by schools</i>	<i>Last date for receipt of solutions from pupils</i>
<i>I</i>	Friday 24 August 2018	Friday 28 September 2018
<i>II</i>	Friday 23 November 2018	Friday 22 February 2019

If there are organisational difficulties you may contact me, Bill Richardson, (wpr3145@gmail.com).

Books of past questions are still available but it seems unlikely that any more will be printed so questions and solutions for 2006-2018 can be accessed at:

[www.wpr3.co.uk/MC-archive/](http://www.wpr3.co.uk/MC-archive/)



# The Scottish Mathematical Council

www.scot-maths.co.uk

## MATHEMATICAL CHALLENGE 2018–2019

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

*CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE*

*The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel,*

*The London Mathematical Society and The Scottish International Education Trust.*

The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

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### Junior Division: Problems 2

**J1.** Each of the digits 2, 3, 5, 7 and 8 is placed one to a box in the diagram.

(a) If the two-digit number is subtracted from the three digit number, what is the smallest possible difference?

(b) If the three-digit number is multiplied by the two-digit number, what is the smallest possible product?

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**J2.** A jeweller makes sets of small cubes out of solid silver. The jeweller has gold-plated none, some, or all of the faces on some of the cubes. The cubes in a set are all different, and no other cube can be added to the set. How many cubes are there in a set?

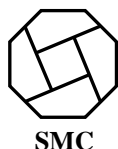
**J3.** Near where I live, there is a very short street of 14 houses numbered 1 to 14 – seven on each side with the odd numbers on one side and the even numbers on the other (numbers 1 and 2 face each other). The really interesting thing about the street is that all the people along one side have names that sound like trades or crafts, while all those on the other side have names which sound like colours.

- Mr Fletcher and Mr Wright live respectively opposite Mr Green and Mr White, who are both neighbours of Mr Black.
- Mr Smith is Mr Mason's father-in-law.
- Mr Mason lives in a higher number than Mr Brown. Mr Mason's and Mr Brown's numbers together equal those of Mr White and Mr Wright together.
- Mrs Taylor's number is twice that of her sister, Mrs Tyler.
- Mr Gray lives opposite to Mr Baker.
- Mrs Tann lives in a double-figure number opposite to her daughter, Mrs Taylor.

What is Mr Scarlett's number?

**J4.** An unlimited supply of petrol is available from a camp at one edge of a desert which is 800 miles wide but no petrol is available anywhere else. A truck can only carry enough petrol to travel 500 miles and is able to leave petrol to be collected later. (There is no limit on the size of such stocks and it should be assumed that no petrol is lost by evaporation or spillage.) Establish whether or not it is possible for the truck to get across the desert and, if it is, explain how.

**SEE OVER FOR QUESTION J5.**



# Mathematical Challenge Problems 2

JUNIOR DIVISION 2018-2019

PLEASE USE CAPITALS TO COMPLETE

SURNAME

OTHER NAME(S)  
(underline the one  
you prefer)

SCHOOL

AGE

YEAR OF STUDY

FOR OFFICIAL USE

Marker

Marks

1	2	3	4	5

Total

— — — — - CUT ALONG HERE — — — —

Please write your solutions on A4 paper and staple the above form to them.

PLEASE WRITE YOUR NAME ON EVERY PAGE.

Send your entry through your school to the section organiser.

For further information on the competition, please see the School Materials which have been distributed to schools. A copy of these Materials can be obtained from <http://www.wpr3.co.uk/MC/materials/index.html>. There are separate links for primary and secondary schools. This page also includes a list of authorities in each section and names and addresses of section organisers.

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- J5.** Two ships, one 200 metres in length and the other 100 metres in length, travel at constant but different speeds. When travelling in opposite directions, it takes 20 seconds for them to completely pass each other. When travelling in the same direction, it takes 50 seconds for them to completely pass each other.

Find the speed of the faster ship.

**END OF PROBLEM SET 2**

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

22 February 2019

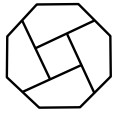
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## MATHEMATICAL CHALLENGE 2018–2019

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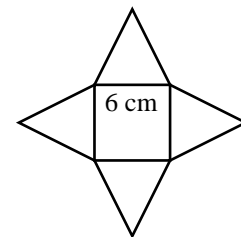
### Middle Division: Problems 2

**M1.** An unlimited supply of petrol is available from a camp at one edge of a desert which is 800 miles wide but no petrol is available anywhere else. A truck can only carry enough petrol to travel 500 miles and is able to leave petrol to be collected later. (There is no limit on the size of such stocks and it should be assumed that no petrol is lost by evaporation or spillage.) Establish whether or not it is possible for the truck to get across the desert and, if it is, explain how.

**M2.** Two ships, one 200 metres in length and the other 100 metres in length, travel at constant but different speeds. When travelling in opposite directions, it takes 20 seconds for them to completely pass each other. When travelling in the same direction, it takes 50 seconds for them to completely pass each other.

Find the speed of the faster ship.

**M3.** Four identical isosceles triangles border a square of side 6 cm, as shown. When the four triangles are folded up they meet at a point to form a pyramid with a square base. If the height of this pyramid is 4 cm, find the total area of the four triangles and the square.



**M4.** Show that the product of four consecutive odd integers is always 16 less than a square number.

Deduce that the product of four consecutive odd integers can never be a square number except in one particular case.

**SEE OVER FOR QUESTION M5.**



# Mathematical Challenge Problems 2

MIDDLE DIVISION 2018-2019

PLEASE USE CAPITALS TO COMPLETE

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OTHER NAME(S) (underline the one you prefer)	<input type="text"/>											
SCHOOL	<input type="text"/>											
AGE	<input type="text"/>	YEAR OF STUDY	<input type="text"/>	S	<input type="text"/>							

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**M5.** A cardboard box manufacturer makes open-topped boxes which are cubes. Because of changes in the market, there are plans to double the volume of the boxes which are made. The regular supplier of raw cardboard offers a 37.5% discount on the price that was originally being charged. A new supplier offers a deal in which the manufacturer would be paying exactly the same for the raw material for his bigger boxes as was paid for the smaller boxes.

Which is the best deal for the manufacturer?

**END OF PROBLEM SET 2**

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

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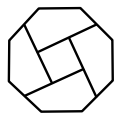
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### Senior Division: Problems 2

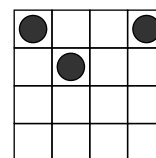
**S1.** Show that the product of four consecutive odd integers is always 16 less than a square number.

Deduce that the product of four consecutive odd integers can never be a square number except in one particular case.

**S2.** A cardboard box manufacturer makes open-topped boxes which are cubes. Because of changes in the market, there are plans to double the volume of the boxes which are made. The regular supplier of raw cardboard offers a 37.5% discount on the price that was originally being charged. A new supplier offers a deal in which the manufacturer would be paying exactly the same for the raw material for his bigger boxes as was paid for the smaller boxes.

Which is the best deal for the manufacturer?

**S3.** In a  $4 \times 4$  grid as shown, place three coins randomly in different squares.



Determine the probability that no two coins lie in the same row or column.

**S4.** Distinct points  $A$ ,  $P$ ,  $Q$ ,  $R$  and  $S$  lie on the circumference of a circle and  $AP$ ,  $AQ$ ,  $AR$  and  $AS$  are chords with the property that

$$\angle PAQ = \angle QAR = \angle RAS.$$

Prove that

$$AR(AP + AR) = AQ(AQ + AS).$$

**SEE OVER FOR QUESTION S5.**



# Mathematical Challenge Problems 2

SENIOR DIVISION 2018-2019

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SURNAME

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(underline the one  
you prefer)

SCHOOL

AGE

YEAR OF STUDY

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- S5.** In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum. Given the magic square shown with all of  $a, b, c, x, y, z$  positive, determine the product  $xyz$  in terms of  $a, b$  and  $c$ .

$\log a$	$\log b$	$\log x$
$p$	$\log y$	$\log c$
$\log z$	$q$	$r$

END OF PROBLEM SET 2

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