Secondary Mathematical Challenges

Welcome to the second round of the 2024-2025 Scottish Secondary Mathematical Challenges. This package contains

This Welcome Page (including Section Information)

Round 2 Questions

In 2024-2025, the name of the Section Organiser is not on the question paper. Their details are on the website but are repeated here for convenience.

Please take great care to use the correct one.

Section 1

Aberdeen City; Aberdeenshire; Highland; Moray; Orkney Islands; Shetland Islands; Western Isles Dr Richard Hepworth (r.hepworth@abdn.ac.uk) Mathematical Challenge Department of Mathematical Sciences, University of Aberdeen, Aberdeen AB24 3UE

Section 2

Angus; Clackmannanshire; Dundee City; Falkirk; Fife; Perth & Kinross; Stirling Dr Jean Reinaud (jnr1@st-andrews.ac.uk) Mathematical Institute, University of St Andrews, St Andrews, Fife KY16 9SS

Section 3

East Lothian; Edinburgh City; Midlothian; Scottish Borders; West Lothian Andrew Gallacher (A.Gallacher@napier.ac.uk) Head of Teacher Education, Edinburgh Napier University, School of Applied Sciences, Room 2.B.37, Sighthill Court, Edinburgh EH11 4BN

Section 4

Argyll & Bute; Dumfries & Galloway; East Ayrshire; East Dunbartonshire; East Renfrewshire; Glasgow City; Inverclyde; North Ayrshire; North Lanarkshire; Renfrewshire; South Ayrshire; South Lanarkshire; West Dunbartonshire Scottish Mathematical Challenge Organiser (wpr3145@gmail.com), Department of Mathematics and Statistics, University of Strathclyde, 26 Richmond Street, Glasgow G1 1XH

The competition timetable for 2024-2025 is as follows:

	Last date for receipt	Last date for receipt
Set No.	of questions by schools	of solutions from pupils
Ι	Friday 23 August 2024	Friday 1 November 2024
II	Friday 22 November 2024	Friday 21 February 2025

If there are organisational difficulties you may contact me, Bill Richardson, (wpr3145@gmail.com).

Books of past questions are still available but it seems unlikely that any more will be printed so questions and solutions for 2006-2018 can be accessed at:

www.wpr3.co.uk/MC-archive/



MC homepage: www.scot-maths.co.uk/

MATHEMATICAL CHALLENGE 2024–2025

A national problem solving competition for schools in Scotland

SECONDARY DIVISIONS

GUIDELINES FOR TEACHERS

1. *Mathematical Challenge* is a problem-solving competition which goes back to 1976. The Challenge is open to all students educated in Scotland. Its aim is to promote mathematics as a source of interest and pleasurable achievement through challenging problems which require only elementary techniques and simple logic.

Please ensure that all teachers involved in the competition see these Guidelines.

How Mathematical Challenge operates

2. There are four divisions: JUNIOR for Sl and S2, MIDDLE for S3 and S4, SENIOR for S5 and S6, and PRIMARY (for which a separate circular is available).

Pupils may enter only one division and must specify that division on their first entry.

Please contact your local organiser, whose name and address are on the proforma on page 2 of each problem sheet and in the Contacts section of the Web pages, if there is any doubt about divisions, or if further information is required.

- 3. There are no written examinations. For the Junior, Middle and Senior Divisions, two sets of five problems each will be available for schools to download according to a timetable outlined in §13 below. Problems for different divisions will be on separate sheets. Some problems may be common to different divisions. The problems will also be available from the Mathematical Challenge Web pages (see above for address).
- 4. A registration fee is required from participating schools. For a secondary school the fee is £20 for the first 10 entrants and £15 for each subsequent batch of 10 entrants or part thereof. A fee form is included with the downloadable pack of materials. For individual participants NOT entering through a school, the fee is £8.

Entries and Marking

- 5. Entries must be the unaided efforts of individual pupils. Group working is not appropriate in Mathematical Challenge. Participants may consult books or the internet for information on facts or on how to tackle problems. Whilst teachers or parents may give guidance on interpretation of wording, they should not be involved in the solution of a problem. Furthermore, the work should not interfere with normal teaching and in no circumstances should it be a class assignment.
- 6. All Sections must use the software package to assist in the processing of the results. A **Record of Entries must be made electronically by the school**, or it will not be possible to process the results.
 - Go to the marks website: https://www.scottishmathschallenge.org.uk/
 - Choose "School Login" and enter your login details or "Register here" to set up a new account.
 - When you have logged in, go to "Add/Edit Entrants" _ enter the names and school year of each entrant.
 - The marks will eventually appear on the "Marks page".
 - Messages from the organiser may also appear there on the first page from time to time.

Use a paper copy of the 'Printable version of details and entrants' from the marks website as a cover sheet for the school's entries. This contains the school details and the alphabetical list of entrants in each section, as entered on the website. All entries submitted will be marked even if earlier problem sets are missed.

- 7. Entries will not be returned. Entrants should keep a copy of their solutions. The Scottish Mathematical Council reserves the right to publish good solutions in its Journal.
- 8. **Participants should explain their solutions as fully as they can**. Marks will be given for explanations of answers not just for the answers themselves. **We should be most grateful if teachers would stress this point**. Incomplete or incorrect answers may gain some credit. In outline, the marking scheme for each problem is as follows:

outline, the marking scheme for each problem is as follow

- 4 : a completely correct solution, with full explanation.
- 3 : a solution, with explanation, which is correct apart from a minor slip or omission of a special case.
- 2: a solution with explanation which contains a serious error or omission, but which nevertheless involves good ideas.
- 1: there is an indication of an interesting idea or method, but not necessarily one which could lead to a correct solution.

A *bonus* mark may be given for a completely correct solution, with full explanation, which contains additional good ideas, such as a successful generalisation of the problem.

A solution in which an answer is given without any explanation will normally be awarded no marks, even if the answer is correct. However, correct working may be accepted as providing an explanation, so long as the various steps are clear.

9. No problems set in *Mathematical Challenge* require the use of a computer package (e.g. a spreadsheet) to obtain a correct solution. If computer software is used, then a proper mathematical explanation of its use is essential.

Awards

- 10. There are three classes of award: *Gold*, *Silver* and *Bronze*. Award winners will be selected primarily on the basis of the total number of marks obtained over both sets of Problems. Special circumstances for individual entrants may be taken into account.
- 11. All award winners will qualify for certificates. Where an award ceremony can be arranged, the most successful entrants will be invited to attend to receive their certificates and Mathematical Challenge mugs. Certificates not presented at a ceremony will be sent by post.

Important notes

- 12. Large numbers of entries can impose a considerable strain on markers and on organisers. Local organisers may have to set limits on the total numbers of entries per school. Schools submitting large numbers of entries may be asked to provide additional markers. Any such markers would not mark entries from their own schools.
- 13. The timetable for 2024-2025 is as follows:

Set No.	Last date for receipt of questions by schools	Last date for receipt of solutions from pupils
Ι	Friday 23 August 2024	Friday 1 November 2024
II	Friday 22 November 2024	Friday 21 February 2025

14. The problems of earlier sessions form an excellent resource. Those for the years 1991-92 to 2005-2006, including solutions, are available in the books *Mathematical Challenges III*, *Mathematical Challenges IV*, *Mathematical Challenges V* and *Mathematical Challenges VI* which are published by The Scottish Mathematical Council. Copies can be obtained from Bill Richardson, Kintail, Longmorn, Elgin IV30 8RJ, prices £7.50, £8, £8, £8 respectively.

In addition, it seems unlikely that any further books will be printed so questions and solutions for 2006-2021 can be accessed at: www.wpr3.co.uk/MC-archive/

Comments on the usefulness of these to wpr3145@gmail.com would be welcome.

15. For other information, please contact your local organiser, whose name and address are given in the Contacts section of the Mathematical Challenge Web pages

www.scot-maths.co.uk

as well as on the materials download menu page

www.wpr3.co.uk/MC/materials



www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2024–2025

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation,

The London Mathematical Society and The Scottish International Education Trust.

The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

Particular thanks are due to the Universities of Aberdeen, Edinburgh Napier, Moray House, St Andrews, Stirling, Strathclyde and to George Heriot's School, Gryffe High School and Kelvinside Academy.

Junior Division: Problems 2

J1. A triangle can be formed with sides of lengths 3, 4 and 6 cm but not with sides of lengths 3, 4 and 7 cm. Oliver has 8 sticks each with length a whole number of cm, but he cannot form a triangle with any 3 of them.

What is the shortest possible length of the longest stick?

J2. In a chemistry lab there are two bottles, each containing a mixture of acid and water:

bottle A contains 140 grams of which 10% is acid,

bottle B contains 60 grams of which 25% is acid.

The lab technician uses some of the mixture from each of the bottles to create a mixture with mass 120 grams of which 15% is acid. Then the lab technician mixes the remaining contents of the bottles to create a new mixture. What percentage of the new mixture is acid?

- **J3.** Three types of item, A, B and C, are for sale. Items of type A sell at 8 for £1. Items of type B sell for £1 each. Items of type C sell for £10 each. A selection of 100 items of all three types costs £100. How many items of type B were there in the selection?
- **J4.** A bag contains 21 balls, each of which is red or blue. The balls are identical except for their colour. Sasha reaches into the bag and removes two balls at random. Each ball in the bag is equally likely to be removed. The probability that two red balls are removed is exactly $\frac{1}{2}$. How many of the 21 balls are red?
- J5. Two joggers live beside a canal. The distance between their houses along the towpath is 5 miles. They each set out at the same time to jog along the towpath to the other's house and back. One jogs at a constant speed of 5 mph and the other is faster with a constant speed of 7 mph. How far from home will the faster jogger be when they meet for the second time? And how long after they set out is this?

END OF PROBLEM SET 2

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

21 February 2025

SEE OVER FOR LINKS TO THE MATHS CHALLENGES ARCHIVES



Mathematical Challenge Problems 2

JUNIOR DIVISION 2024-2025

SURNAME		FOR OFFICIAL USE Marker
OTHER NAME(S) (underline the one you prefer)		Marks
SCHOOL		
AGE	YEAR OF STUDY S	Total

- CUT ALONG HERE--

Please write your solutions on A4 paper and staple the above form to them. PLEASE WRITE YOUR NAME ON EVERY PAGE. Send your entry through your school to the section organiser.

For further information on the competition, please see the School Materials which have been distributed to schools. A copy of these Materials can be obtained from

http://www.wpr3.co.uk/MC/materials/index.html

There are separate links for primary and secondary schools. This page also includes a list of authorities in each section and names and addresses of section organisers.

For information about Mathematical Challenge, look on the SMC web site:

www.scot-maths.co.uk

MATHS CHALLENGES ARCHIVES

There are archives of previous questions on: www.wpr3.co.uk/MC-archive/J/index-J.html

Here is a shortcut for your smartphone or tablet





www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2024–2025

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation,

The London Mathematical Society and The Scottish International Education Trust.

The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

Particular thanks are due to the Universities of Aberdeen, Edinburgh Napier, Moray House, St Andrews, Stirling, Strathclyde and to George Heriot's School, Gryffe High School and Kelvinside Academy.

Middle Division: Problems 2

- M1. A bag contains 21 balls, each of which is red or blue. The balls are identical except for their colour. Sasha reaches into the bag and removes two balls at random. Each ball in the bag is equally likely to be removed. The probability that two red balls are removed is exactly $\frac{1}{2}$. How many of the 21 balls are red?
- M2. Two joggers live beside a canal. The distance between their houses along the towpath is 5 miles. They each set out at the same time to jog along the towpath to the other's house and back. One jogs at a constant speed of 5 mph and the other is faster with a constant speed of 7 mph. How far from home will the faster jogger be when they meet for the second time? And how long after they set out is this?
- **M3.** On Saturday I cycled from my home due east to the junction at Aton. I continued due east (though not as far as before) to the junction at Beesley. I returned home via the junction at Ceeborough, which is due north along a straight road from Aton. All the roads are straight and it is a whole number of miles between any neighbouring junctions. My journey was less than 48 miles. What distance did I cycle?
- M4. Around the circumference of a circle, mark 21 points, equally spaced, and label them 0, 1, 2, ..., 20 in cyclic order. Mark n of the points with an asterisk (*) so that no two pairs of * points are the same distance apart. What is the maximum value of n? Explain.
- **M5.** Find a set of 4 different positive integers such that the sum of any pair of them divides their product exactly.

Find a set of 6 different positive integers with the same property, or show that it is not possible to find such a set.

END OF PROBLEM SET 2

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

21 February 2025

SEE OVER FOR LINKS TO THE MATHS CHALLENGES ARCHIVES



Mathematical Challenge Problems 2

MIDDLE DIVISION 2024-2025

PLEASE USE CAPITALS TO COMPLETE

SURNAME		FOR OFFICIAL USE Marker
OTHER NAME(S) (underline the one you prefer)		Marks
SCHOOL		
AGE	YEAR OF STUDY S	Total

- CUT ALONG HERE - -

Please write your solutions on A4 paper and staple the above form to them. PLEASE WRITE YOUR NAME ON EVERY PAGE.

Send your entry through your school to the section organiser.

For further information on the competition, please see the School Materials which have been distributed to schools. A copy of these Materials can be obtained from

http://www.wpr3.co.uk/MC/materials/index.html

There are separate links for primary and secondary schools. This page also includes a list of authorities in each section and names and addresses of section organisers.

LINKS TO THE MATHS CHALLENGES ARCHIVES

There are archives of previous questions on: www.wpr3.co.uk/MC-archive/M/index-M.html

Here is a shortcut for your smartphone or tablet





www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2024–2025

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation,

The London Mathematical Society and The Scottish International Education Trust.

The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

Particular thanks are due to the Universities of Aberdeen, Edinburgh Napier, Moray House, St Andrews, Stirling, Strathclyde and to George Heriot's School, Gryffe High School and Kelvinside Academy.

Senior Division: Problems 2

- **S1.** Around the circumference of a circle, mark 21 points, equally spaced, and label them 0, 1, 2, ..., 20 in cyclic order. Mark *n* of the points with an asterisk (*) so that no two pairs of * points are the same distance apart. What is the maximum value of *n*? Explain.
- **S2.** Find a set of 4 different positive integers such that the sum of any pair of them divides their product exactly.

Find a set of 6 different positive integers with the same property, or show that it is not possible to find such a set.

S3. Triangle *ABC* is right-angled at *C* and has side lengths which are integers. A second triangle, *PQR*, is located inside $\triangle ABC$ as shown, such that its sides are parallel to the sides of $\triangle ABC$ and the distance between all pairs of parallel lines is 2. Determine the side lengths of all possible triangles *ABC*, so that the area of $\triangle ABC$ is 9 times the area of $\triangle PQR$.



S4.



QRS is a straight line. Triangles PQR and RST are equilateral. M is the midpoint of PS and N is the midpoint of TQ.

Prove that triangle *MNR* is equilateral.

SEE OVER FOR QUESTION S5.



Mathematical Challenge Problems 2

SENIOR DIVISION 2024-2025

PLEASE USE CAPITALS TO COMPLETE

SURNAME		FOR OFFICIAL USE
OTHER NAME(S) (underline the one you prefer)		Marks
SCHOOL		
AGE	YEAR OF STUDY S	Total

- CUT ALONG HERE -

Please write your solutions on A4 paper and staple the above form to them. PLEASE WRITE YOUR NAME ON EVERY PAGE. Send your entry through your school to the section organiser.

For further information on the competition, please see the School Materials which have been distributed to schools. A copy of these Materials can be obtained from

http://www.wpr3.co.uk/MC/materials/index.html

There are separate links for primary and secondary schools. This page also includes a list of authorities in each section and names and addresses of section organisers.

S5. Determine all real solutions of the pair of equations:

$$\sqrt{a} + b = 8$$

$$\log_{10} a + 2 \log_{10} b = 2$$

Give your answer(s) as simplified exact numbers.

END OF PROBLEM SET 2

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

21 February 2025

For information about Mathematical Challenge, look on the SMC web site: www.scot-maths.co.uk

MATHS CHALLENGES ARCHIVES

There are archives of previous questions on: www.wpr3.co.uk/MC-archive/S/index-S.html Here is a shortcut for your smartphone or tablet

