Secondary Mathematical Challenges

Welcome to the 2023-2024 Scottish Secondary Mathematical Challenges. This package contains

> This Welcome Page (including Section Information)
> The Secondary Guidelines
> Round 1 Questions
> Information About Payments
> Data Protection and Privacy Policy
> How to Enter your School and Pupil Names on the Marks Website
> The Annual Poster
> Archives and Book Order Form

Once again, the name of the Section Organiser is not on the question paper. Their details are on the website but are repeated here for convenience:

Section 1

Aberdeen City; Aberdeenshire; Highland; Moray; Orkney Islands; Shetland Islands; Western Isles

Dr Richard Hepworth (r.hepworth@abdn.ac.uk)
Mathematical Challenge
Department of Mathematical Sciences, University of Aberdeen,
Aberdeen AB24 3UE

Section 2

Angus; Clackmannanshire; Dundee City; Falkirk; Fife; Perth & Kinross; Stirling

Dr Jean Reinaud (jnr1@st-andrews.ac.uk)
Mathematical Institute, University of St Andrews,
St Andrews, Fife KY16 9SS

Section 3

East Lothian; Edinburgh City; Midlothian; Scottish Borders; West Lothian

Andrew Gallacher (A.Gallacher@napier.ac.uk)
Head of Teacher Education, Edinburgh Napier University, School of Applied Sciences,
Room 2.B.37, Sighthill Court, Edinburgh EH11 4BN


Section 4

Argyll & Bute; Dumfries & Galloway; East Ayrshire; East Dunbartonshire;
East Renfrewshire; Glasgow City; Inverclyde; North Ayrshire; North Lanarkshire;
Renfrewshire; South Ayrshire; South Lanarkshire; West Dunbartonshire

Scottish Mathematical Challenge Organiser (wpr3145@gmail.com),
Department of Mathematics and Statistics, University of Strathclyde,
26 Richmond Street, Glasgow G1 1XH

The competition timetable for 2023-2024 is as follows:

Set No.	*Last date for receipt of questions by schools*	*Last date for receipt of solutions from pupils*
I	Friday 25 August 2023	Friday 3 November 2023
II	Friday 24 November 2023	Friday 23 February 2024

If there are organisational difficulties you may contact me, Bill Richardson, (wpr3145@gmail.com).
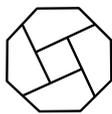
The Scottish Mathematical Council

MC homepage: www.scot-maths.co.uk/

MATHEMATICAL CHALLENGE 2023–2024

A national problem solving competition for schools in Scotland

S E C O N D A R Y   D I V I S I O N S

GUIDELINES FOR TEACHERS

1. *Mathematical Challenge* is a problem-solving competition which goes back to 1976. The Challenge is open to all students educated in Scotland. Its aim is to promote mathematics as a source of interest and pleasurable achievement through challenging problems which require only elementary techniques and simple logic.
   Please ensure that all teachers involved in the competition see these Guidelines.

How Mathematical Challenge operates

2. There are four divisions: JUNIOR for Sl and S2, MIDDLE for S3 and S4, SENIOR for S5 and S6, and PRIMARY (for which a separate circular is available).
   Pupils may enter only one division and must specify that division on their first entry.
   Please contact your local organiser, whose name and address are on the proforma on page 2 of each problem sheet and in the Contacts section of the Web pages, if there is any doubt about divisions, or if further information is required.

3. There are no written examinations. For the Junior, Middle and Senior Divisions, two sets of five problems each will be available for schools to download according to a timetable outlined in §13 below. Problems for different divisions will be on separate sheets. Some problems may be common to different divisions. The problems will also be available from the Mathematical Challenge Web pages (see above for address).

4. **A registration fee is required from participating schools. For a secondary school the fee is £20 for the first 10 entrants and half this amount for each subsequent batch of 10 entrants or part thereof. A fee form is included with the downloadable pack of materials.**
   For individual participants NOT entering through a school, the fee is £8.

Entries and Marking

5. **Entries must be the unaided efforts of individual pupils. Group working is not appropriate in Mathematical Challenge.** Participants may consult books or the internet for information on facts or on how to tackle problems. Whilst teachers or parents may give guidance on interpretation of wording, **they should not be involved in the solution of a problem.** Furthermore, **the work should not interfere with normal teaching and in no circumstances should it be a class assignment.**

6. All Sections must use the software package to assist in the processing of the results. A **Record of Entries must be made electronically by the school**, or it will not be possible to process the results.

   • Go to the marks website: https://www.scottishmathschallenge.org.uk/
   • Choose "School Login" and enter your login details or "Register here" to set up a new account.
   • When you have logged in, go to "Add/Edit Entrants" _ enter the names and school year of each entrant.
   • The marks will eventually appear on the "Marks page".
   • Messages from the organiser may also appear there on the first page from time to time.

   Use a paper copy of the 'Printable version of details and entrants' from the marks website as a cover sheet for the school's entries. This contains the school details and the alphabetical list of entrants in each section, as entered on the website. All entries submitted will be marked even if earlier problem sets are missed.

7. Entries will not be returned. Entrants should keep a copy of their solutions. The Scottish Mathematical Council reserves the right to publish good solutions in its Journal.

8. **Participants should explain their solutions as fully as they can**. Marks will be given for explanations of answers not just for the answers themselves. **We should be most grateful if teachers would stress this point**. Incomplete or incorrect answers may gain some credit.

   In outline, the marking scheme for each problem is as follows:

   4 : a completely correct solution, with full explanation.
   3 : a solution, with explanation, which is correct apart from a minor slip or omission of a special case.
   2 : a solution with explanation which contains a serious error or omission, but which nevertheless involves good ideas.
   1 : there is an indication of an interesting idea or method, but not necessarily one which could lead to a correct solution.

   A *bonus* mark may be given for a completely correct solution, with full explanation, which contains additional good ideas, such as a successful generalisation of the problem.

   A solution in which an answer is given without any explanation will normally be awarded no marks, even if the answer is correct. However, correct working may be accepted as providing an explanation, so long as the various steps are clear.

9. No problems set in *Mathematical Challenge* require the use of a computer package (e.g. a spreadsheet) to obtain a correct solution. If computer software is used, then a proper mathematical explanation of its use is essential.

Awards

10. There are three classes of award: *Gold*, *Silver* and *Bronze*. Award winners will be selected primarily on the basis of the total number of marks obtained over both sets of Problems. Special circumstances for individual entrants may be taken into account.

11. All award winners will qualify for certificates. Where an award ceremony can be arranged, the most successful entrants will be invited to attend to receive their certificates and Mathematical Challenge mugs. Certificates not presented at a ceremony will be sent by post.

Important notes

12. Large numbers of entries can impose a considerable strain on markers and on organisers. Local organisers may have to set limits on the total numbers of entries per school. Schools submitting large numbers of entries may be asked to provide additional markers. Any such markers would not mark entries from their own schools.

13. The timetable for 2022-2023 is as follows:

Set No.	Last date for receipt of questions by schools	Last date for receipt of solutions from pupils
I	Friday 25 August 2023	Friday 3 November 2023
II	Friday 24 November 2023	Friday 23 February 2024

14. **The problems of earlier sessions form an excellent resource.** Those for the years 1991-92 to 2005-2006, including solutions, are available in the books *Mathematical Challenges III*, *Mathematical Challenges IV*, *Mathematical Challenges V* and *Mathematical Challenges VI* which are published by The Scottish Mathematical Council. Copies can be obtained from Bill Richardson, Kintail, Longmorn, Elgin IV30 8RJ, prices £7.50, £8, £8, £8 respectively.

   In addition, it seems unlikely that any further books will be printed so questions and solutions for 2006-2021 can be accessed at: **www.wpr3.co.uk/MC-archive/**

   Comments on the usefulness of these to wpr3145@gmail.com would be welcome.

15. For other information, please contact your local organiser, whose name and address are given in the Contacts section of the Mathematical Challenge Web pages

   www.scot-maths.co.uk

   as well as on the materials download menu page

   www.wpr3.co.uk/MC/materials

Junior Division: Problems 1

J1. Six integers have a mean of 32. The median is $33\frac{1}{2}$. The mode is 35, which is not the largest integer. The range is 11. What are the possible sets of 6 integers?

J2. One of the highlights of the local village social life is the stage production organised by the Amateur Youth Players and the rehearsals are in full swing for *The Gondoliers*. When I called the treasurer the other day he was estimating the costs. The first scene, in case you have forgotten, shows 24 maidens of Venice making up small bunches of red and white roses. He had intended that each girl would have three red and two white roses until he realised that the red roses cost twice as much each as the white ones. He decided to give half the girls three red and two white roses each and the remainder two red and three white roses. He had cut the cost by £3. How much is a red rose?

J3. At the secondary school down the road Chemistry, English, French, Geography, Maths and Physics are taught by Mr Brown, Mr Jones, Mrs Robinson and Ms Singh.

Each teacher teaches three subjects and each subject is taught by two teachers.

Two of Ms Singh's subjects are also taught by Mr Jones.

Maths is shared by Ms Singh and Mrs Robinson.

Both of the teachers of English also teach French.

Mrs Robinson teaches Chemistry and Mr Jones does not teach Physics.

Who are the two teachers of Geography?

J4.
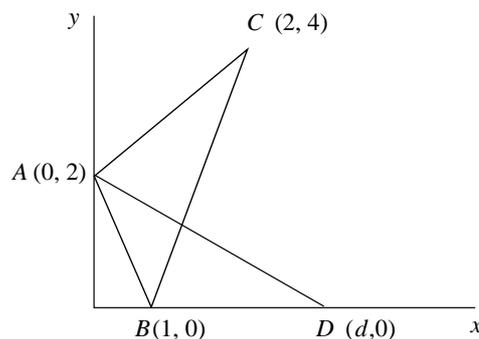
In the diagram, $D(d, \ 0)$ lies on the x-axis beyond B. The triangles ABC and ABD have the same area. Determine the value of d.

SEE OVER FOR QUESTION J5.

Mathematical Challenge Problems 1

SMC

JUNIOR DIVISION 2023-2024
PLEASE USE CAPITALS TO COMPLETE

– – – – - C U T   A L O N G   H E R E – – – –

J5. "Will those in favour of the resolution please hold up their hands?" said the chairperson at a public meeting.

On a count of hands, it appeared that the resolution was carried by a majority of 7. It was then found that, in the excitement of the moment, 6% of those in favour were holding up both their hands. When this had been allowed for, the actual result of the vote proved to be a majority of 2 against the resolution.

How many people voted?

END OF PROBLEM SET 1
CLOSING DATE FOR RECEIPT OF SOLUTIONS :

3 November 2023

Look out for Problems 2 in late November!

For information about Mathematical Challenge, look on the SMC web site:

www.scot-maths.co.uk

The Scottish Mathematical Council

MATHEMATICAL CHALLENGE 2023–2024

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.
CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
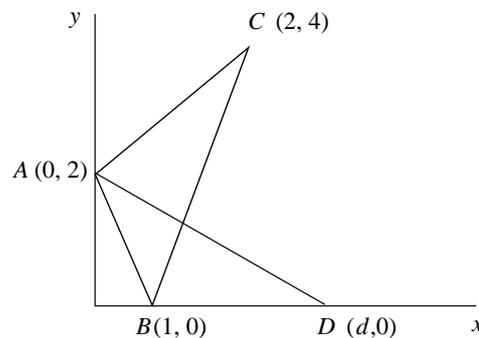***The Edinburgh Mathematical Society, The Maxwell Foundation,***
The London Mathematical Society and The Scottish International Education Trust.

Middle Division: Problems 1

M1.

In the diagram, $D(d, 0)$ lies on the x-axis beyond B. The triangles ABC and ABD have the same area. Determine the value of d.

M2. "Will those in favour of the resolution please hold up their hands?" said the chairperson at a public meeting.

On a count of hands, it appeared that the resolution was carried by a majority of 7. It was then found that, in the excitement of the moment, 6% of those in favour were holding up both their hands. When this had been allowed for, the actual result of the vote proved to be a majority of 2 against the resolution.

How many people were asked to record their vote?

M3. The distance from St Andrews to Leven is 20 miles by one road and 24 miles by another. A cyclist uses one road going out and the other coming back but whichever way she goes her return journey is 2 mph slower than the outward journey. She also found that if she goes out by the longer road and returns on the shorter she takes 6 minutes less time than if she went the other way round.

What is her speed on the outward journey?

SEE OVER FOR QUESTIONS M4 & M5.

Mathematical Challenge Problems 1

MIDDLE DIVISION 2023-2024

PLEASE USE CAPITALS TO COMPLETE

SMC

SURNAME

OTHER NAME(S)
(underline the one
 you prefer)

SCHOOL

AGE   YEAR OF STUDY S

— — — — CUT ALONG HERE — — — —

Please write your solutions on A4 paper and staple the above form to them.
PLEASE WRITE YOUR NAME ON EVERY PAGE.
Send your entry through your school to the section organiser.

For further information on the competition, please see the School Materials which have been distributed to schools. A copy of these Materials can be obtained from

http://www.wpr3.co.uk/MC/materials/index.html

There are separate links for primary and secondary schools. This page also includes a list of authorities in each section and names and addresses of section organisers.

M4. In the diagram, $2\angle BAC = 3\angle ABC$ and D lies on BC such that $\angle DAC = 2\angle DAB$. Suppose that $BC = a, AC = b, AB = c, AD = d$ and $CD = e$.
Find expressions for d and e in terms of a, b and c only.
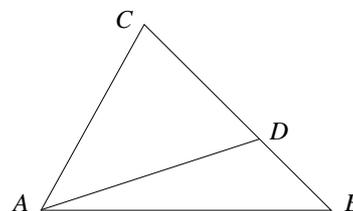
M5. A school assembly hall has a rectangular array of chairs. There are exactly 12 boys seated in each of the r rows and exactly 10 girls seated in each of the c columns. There are fewer than 1000 boys and girls in the school. There is just one empty chair.

How many chairs are there in the assembly hall?
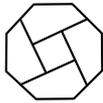
END OF PROBLEM SET 1

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

3 November 2023

Look out for Problems 2 in late November!

For information about Mathematical Challenge, look on the SMC web site:
www.scot-maths.co.uk

The Scottish Mathematical Council

MATHEMATICAL CHALLENGE 2023–2024

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

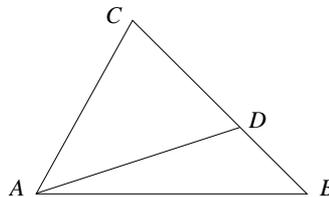***The Edinburgh Mathematical Society, The Maxwell Foundation,***

The London Mathematical Society and The Scottish International Education Trust.

Senior Division: Problems 1

S1.

In the diagram, $2\angle BAC = 3\angle ABC$ and D lies on BC such that $\angle DAC = 2\angle DAB$. Suppose that $BC = a, AC = b, AB = c, AD = d$ and $CD = e$.
Find expressions for d and e in terms of a, b and c only.

S2. A school assembly hall has a rectangular array of chairs. There are exactly 12 boys seated in each of the r rows and exactly 10 girls seated in each of the c columns. There are fewer than 1000 boys and girls in the school. There is just one empty chair.

How many chairs are there in the assembly hall?

S3.
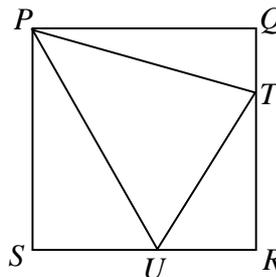
In the diagram, *PQRS* is a square with sides of length 2. Points *T* and *U* are on sides *QR* and *RS* respectively such that $\angle TPU = 45°$.
Determine the minimum possible perimeter of triangle *RTU*.

S4. George throws three unbiased dice and removes all of the dice that come up with a 5 or 6. Martha then throws the dice that remain, if any. Determine the probability that exactly one of Martha's dice shows a 5 or 6.

SEE OVER FOR QUESTION S5.

Mathematical Challenge Problems 1

SENIOR DIVISION 2023-2024
PLEASE USE CAPITALS TO COMPLETE

SMC

S5. The irrational number $\sqrt{2}$ can be written as a series of continued fractions in the following way

$$\sqrt{2} = 1 + (\sqrt{2} - 1) = 1 + \frac{1}{\sqrt{2} + 1}$$

$$= 1 + \frac{1}{2 + (\sqrt{2} - 1)} = 1 + \frac{1}{2 + \frac{1}{\sqrt{2} + 1}}$$

$$= 1 + \frac{1}{2 + \frac{1}{2 + (\sqrt{2} - 1)}} = 1 + \frac{1}{2 + \frac{1}{2 + \frac{1}{\sqrt{2} + 1}}}$$

This process can be continued. If we stop after n steps and ignore the term containing $\sqrt{2}$ we get a rational number $\frac{p_n}{q_n}$. So

$$\frac{p_1}{q_1} = 1, \qquad \frac{p_2}{q_2} = 1 + \frac{1}{2} = \frac{3}{2}, \qquad \frac{p_3}{q_3} = 1 + \frac{1}{2 + \frac{1}{2}} = \frac{7}{5}$$

and so on.

Show that, for all odd integers n, $\frac{p_n}{q_n} < \sqrt{2}$ and for all even n, $\frac{p_n}{q_n} > \sqrt{2}$.
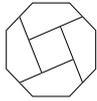
END OF PROBLEM SET 1

CLOSING DATE FOR RECEIPT OF SOLUTIONS :

| **3 November 2023** |

Look out for Problems 2 in late November!

For information about Mathematical Challenge, look on the SMC web site:
www.scot-maths.co.uk

The Scottish Mathematical Council

MATHEMATICAL CHALLENGE (SECONDARY)

Registration Fee 2023–2024

Participating schools are required to pay a registration fee. The fee covers both sets of problems. The secondary school fee is £20 for 1-10 entrants in Round 1, £10 for the next 10 (or part thereof) and so on. A cheque made payable to

SMC Mathematical Challenge

should be attached to this form (but see below) and

returned with the first set of problems.

School details	Name	. . . .
	Telephone	. . . .
	Address	. . . .
		. . . .
		. . . .
		. . . .
	Education Authority (where appropriate)	. . . .
Contact teacher	Name	. . . .
	email address (if used)	. . . .

Payments: we prefer to be paid by cheque but if this is not possible we will accept payment by BACS. Our account is:

SMC MATHEMATICAL CHALLENGE

The details are: **83-06-08 1054 8147.**

The reference you provide **must** include the school postcode followed by the school name.

Data Protection

Our policy is attached. By completing and returning this form we will assume that you have read it and accept it.

Submission of scripts

You must submit **on time** and use **correct postage**. As a guide, first class large letters rates are:

Weight below	100g	250g	500g	750g
Postage	£1.60	£2.25	£2.95	£3.30

Data Protection and Privacy Policy for the SMC Maths Challenge

In the following, the data referred to is stored on the computer used for entering pupil and school details and for recording pupil performance. Access to all such information is password protected.

For each **entrant** the data stored is:

- surname, other name, school, year of study, and (after marking) the marks for each question. Permission to record this data is given by completing the tear-off form attached to the solutions.

For each **school** the data stored is:

- school name, address, phone, Primary or Secondary, LEA (Local Education Authority area), whether independent, contact teacher name and e-mail, and other information you want to add (if any).

Teachers have access, by logging in with their username and password, only to the data for their own school and its entrants, and are responsible for communicating results and awards to their entrants.

Section Organisers have access, by logging in with their username and password, only to data about schools and entrants within their own Section.

Designated members of the Maths Challenge National Committee have access, by logging in with their username and password, to all the data in the database to facilitate running the challenge.

No data will be kept for more than 4 years. This would allow for production of comparative reports over 3 years for the Scottish Mathematical Council Journal.

The data is stored on a secure server.

Contact: *scottishmathschallenge@gmail.com*

Using the marks website

A Record of Entrants must be made on the marks website by the school or it will not be possible for the section organiser to later add the marks for each entrant.

Go to the website:

> https://www.scottishmathschallenge.org.uk/

If you have used the website for your school in a previous year and can remember the username and password log in as usual.

Otherwise choose

> "Register here"

which enables you to enter your details and create a password. Your username will be your email address so please make sure that you enter this correctly. The system will send you an email with an activation link: click on this link to verify your email address and then log in.

You will need your username and password each time you log in.

Once logged in click on the link

> "Edit School Details"

Check that all the contact details are correct, making changes as necessary. In particular, make sure that you select primary or secondary and the relevant local authority whenever you make changes.

Later, when you have a pile of entries, or a list of entrants, log in again and go to

> "Add/Edit Entrants"

and enter
- the division (only for secondary pupils),
- the names, and
- the school year of each entrant.

When all your entrants have been added, to get a printout to send with your entries go to

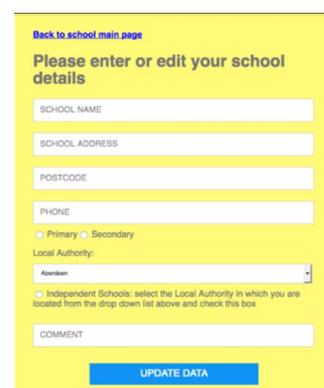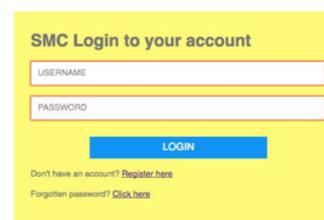> "Printable version of details and entrants"

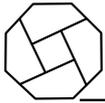When the marks for each round are released, to view the marks for your school go to

> "Marks page"

On the main page after you log in you may, from time to time, see messages from the section organisers.

NOTE: If you experience difficulties with the website please contact either

> your section organiser or
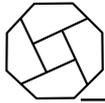> Helen Martin h.martin@abdn.ac.uk

MATHEMATICAL CHALLENGE 2023–2024

One of the highlights of the local village social life is the stage production organised by the Amateur Youth Players and the rehearsals are in full swing for *The Gondoliers*. When I called the treasurer the other day he was estimating the costs. The first scene, in case you have forgotten, shows 24 maidens of Venice making up small bunches of red and white roses. He had intended that each girl would have three red and two white roses until he realised that the red roses cost twice as much each as the white ones. He decided to give half the girls three red and two white roses each and the remainder two red and three white roses. He had cut the cost by £3. How much is a red rose?

The Scottish Mathematical Council
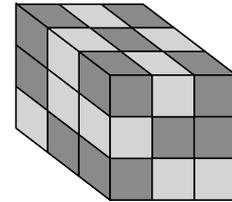
www.scot-maths.co.uk

MATHEMATICAL CHALLENGE
2022–2023
Did you solve it?

Twenty-seven unit cubes are each coloured completely blue or completely yellow. All twenty-seven unit cubes are assembled into a larger cube. If half of the surface area of the larger cube is blue, what is the largest number of unit cubes that could have been coloured blue?
The surface area of one face of a 3 x 3 unit cube is 9 square units.

Solution

There are 6 faces, so 54 square units in all.

The largest number of blue cubes occurs with the smallest number of yellow cubes, and it's easier to count fewer cubes.

One half of the surface area is yellow, i.e. $\frac{1}{2} \times 54 = 27$ unit faces.

The greatest area of yellow from a unit yellow cube is when it is in a corner of the large cube, which would supply 3 unit faces. We require 27 unit faces, so putting yellow cubes in all 8 corners will account for 24 of the yellow unit faces.

The next greatest area of yellow from a unit yellow cube is when it is in the middle of an edge of the large cube, which would supply 2 unit faces. So we need one edge unit cube to take the total to 26 yellow unit faces.

The final unit face comes from a yellow cube in the centre of a face of the large cube. This gives 10 yellow cubes.

(The numbers in the diagram below show how many faces of a yellow cube can be seen.)

	top				middle				bottom		
3	2	3							3		3
	1										
3		3							3		3

A corner yellow cube and the yellow cube in the centre of a face could be replaced by 2 edge yellow cubes, but that is still a total of 10 yellow cubes.

So there is a minimum of 10 unit cubes coloured yellow and hence a maximum of $27 - 10 = 17$ unit cubes coloured blue.

Mathematical Challenges

The Scottish Mathematical Council
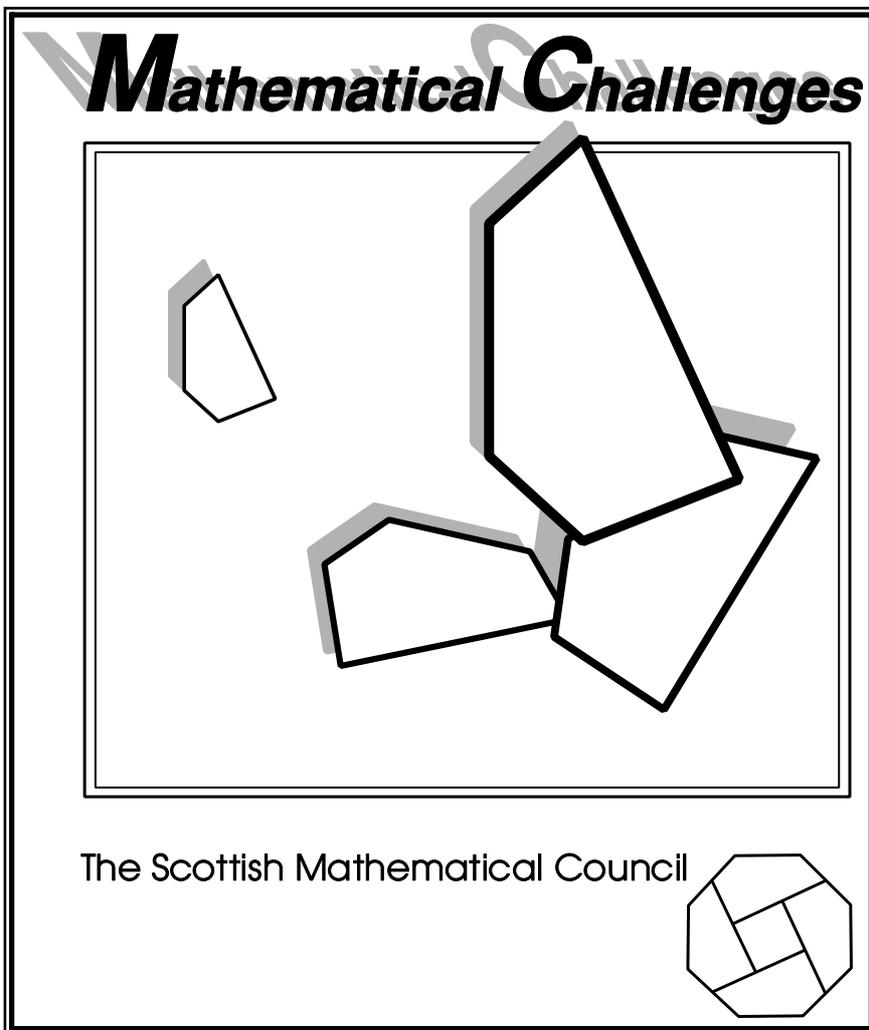
Archives and Books

The books listed below and there are still limited stocks. However, since 2006, an online archive has been developed. It covers all Challenges, with sheets with questions and separate sheets which include solutions. These are all avaialble at

www.wpr3.co.uk/MC-archive/

The books which were puplisehd are as follows:

		Price	Number	Cost
Mathematical Challenges VI	2003 to 2006	£8.00		
Mathematical Challenges V	2000 to 2003	£8.00		
Mathematical Challenges IV	1997 to 2000	£8.00	_____	_____
Mathematical Challenges III	1994 to 1997	£7.50	_____	_____
Primary Mathematical Challenges	up to 2001	£5.00	_____	_____
		Total		_____

Copies of any of these can be obtained from

   Bill Richardson, Kintail, Longmorn, Elgin IV30 8RJ

Cheques with order are preferred.

Cheques should be made payable to 'SMC Mathematical Challenge'.

A few copies of Mathematical Challenges II and also the original
Blackie Mathematical Challenges are still available.

email enquiries:- wpr3145@gmail.com