The Scottish Mathematical Council

www.scot-maths.co.uk

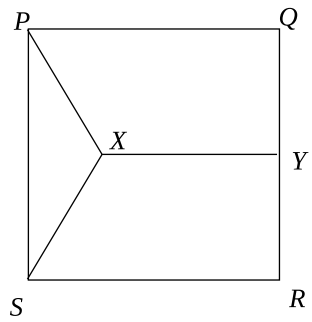
MATHEMATICAL CHALLENGE 2020−2021

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

Senior Division: Problems 2

S1. A large equilateral triangle with sides of integer length N is split into small equilateral triangular cells each with side length 1 by drawing lines parallel to its sides. A continuous track starts in the cell at one corner of the large triangle and moves from cell to cell, always crossing at an edge shared by the two cells. The track never revisits a cell. Find, with proof, the greatest number of cells that can be visited on one track.

S2. In the diagram, $PQRS$ is a square with XY perpendicular to QR and $XP = XS = XY = 10$ cm. What is the area of the square?



S3. If x is a real number satisfying $x^3 + \frac{1}{x^3} = 2\sqrt{5}$, determine the exact value of $x^2 + \frac{1}{x^2}$.

Hint: Start by looking at $\left(x + \frac{1}{x}\right)^2$.

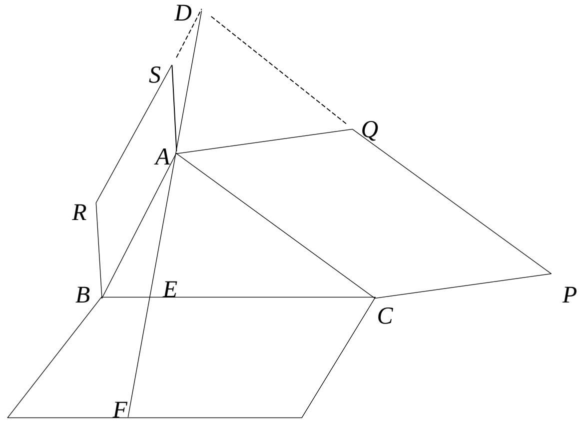
S4. x and y are positive integers such that $x^2 + y^2 - x$ is exactly divisible by $2xy$.

(a) Find all possible values for y when $x = 9$.

(b) Show that x must always be a perfect square.

S5. Triangle BAC has a right angle at A. Any two parallelograms, $ACPQ$ and $ABRS$ are constructed on AC and AB respectively. The lines PQ and RS are produced to meet at D. The line $DAEF$ is drawn with $DA = EF$.

Show that the area of any parallelogram with side BC and F lying on the opposite side equals the sum of the areas of the parallelograms $ABRS$ and $ACPQ$.



END OF PROBLEM SET 2