



The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2020–2021

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel,

The London Mathematical Society and The Scottish International Education Trust.

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Senior Division: Problems 1

- S1.** A parent has washed some nappies in a strong bleach solution and wishes to rinse them so that they contain as weak a bleach solution as possible. By wringing out, the nappies can be made to contain just half a litre of solution. Show that two thorough rinses, such that the solution strength is uniform, the first using 12 litres of water and the second using 8 litres of water, reduces the strength of the bleach solution to $\frac{1}{425}$ of its original value.

If 20 litres of clean water is all that is available and the parent is prepared to do only two rinses, how best should the water be divided between the two rinses?

- S2.** A pyramid has a square base and four equilateral triangles as its other faces. The four equilateral triangles can also make a tetrahedron. What is the ratio of the volumes of the pyramid and the tetrahedron? **Justify your answer.**
- S3.** $ABCD$ is a cyclic quadrilateral. BD bisects $\angle ABC$. Extend the side BA beyond A to a point E . Show that $DE = DB$ if and only if $AE = BC$.
- S4.** A coin is biased so that the probability of obtaining a head is p , where $0 < p < 1$. If the sequence HHH occurs first then Player A wins, and if the sequence HTH occurs first then player B wins. The coin is tossed until one player wins. For what value of p is the game fair?
(A fair game is one in which both players have an equal chance of winning.)
- S5.** Given that $f(x) = x^2 - x - 1$, $g(x) = ax + b$ and $f(g(x)) = 4x^2 - 10x + 5$, determine all possible pairs of values of a and b which satisfy this relationship.

END OF PROBLEM SET 1