The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2019−2020

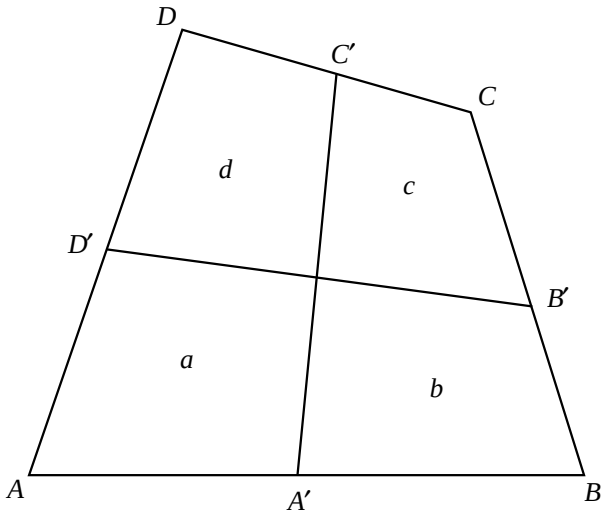
Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

Senior Division: Problems 2

S1. A positive integer ends in the digit 4 and has the property that it becomes four times as large when the 4 is moved from the end and placed at the front. What is the smallest such number?

S2.

Let $ABCD$ be a quadrilateral. Let A' be the midpoint of AB, B' the mid-point of BC, C' the mid-point of CD and D' the mid-point of AD. Draw the lines $A'C'$ and $B'D'$ and let a, b, c, d be the areas of the four minor quadrilaterals as shown in the figure. Prove that
$a + c = b + d$.

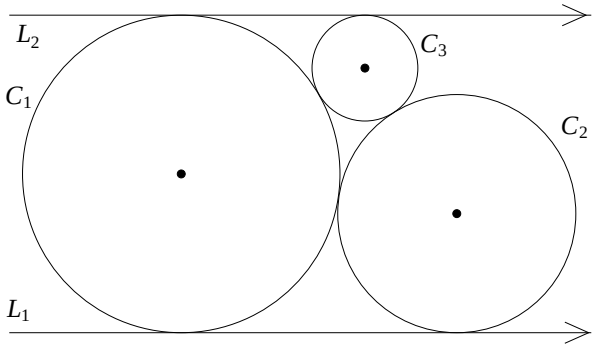


S3. Find all values of x such that

$$\log_2(3x + 2) + \log_2(4x - 4) = 3.$$

S4.

The diagram shows three circles, C_1, C_2 and C_3 and two parallel lines, L_1 and L_2. C_1 touches both the lines, C_2 touches L_1 and C_1, and C_3 touches C_1, C_2 and L_2. The radius of C_2 is 16 and the radius of C_3 is 9. Find the radius of C_1.



S5. If p and q are positive integers, $\max(p, q)$ is the maximum of p and q and $\min(p, q)$ is the minimum of p and q. So for example $\max(3, 6) = 6$ and $\min(3, 6) = 3$.

Determine the number of ordered pairs (x, y) which satisfy the equation

$$\max\big(70, \min(x, y)\big) = \min\big(\max(70, x), y\big)$$

where x and y are positive integers with $x \leqslant 100$ and $y \leqslant 100$.

END OF PROBLEM SET 2