The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2019−2020

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel, The London Mathematical Society and The Scottish International Education Trust.

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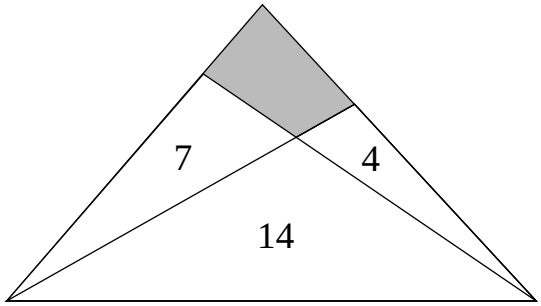
Senior Division: Problems 1

S1. There are 10 lockers in a row, numbered from 1 to 10. Each locker is to be painted red or blue or green, subject to the following rules:

- two lockers with numbers n and m are painted different colours whenever $n - m$ is odd;
- it is not necessary to use all 3 colours.

In how many different ways can the row of lockers be painted? Justify your answer.

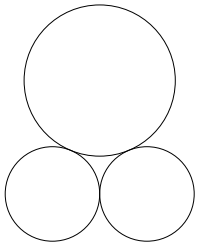
S2. In the diagram (which is not drawn to scale) the small triangles each have the area shown. Find the area of the shaded quadrilateral.



S3. Two circular discs of radius 5 cm and one circular disc of radius 8 cm are placed flat on a table with their edges touching.

(a) Determine the exact radius of the largest disc that can fit in the space between these three discs.

(b) Determine the exact radius of the smallest disc that can surround these three discs.



S4. Find all solutions of the pair of equations

$$x^2 + x^2y^2 + x^2y^4 = 525$$

and

$$x + xy + xy^2 = 35.$$

S5. The parabola $y = x^2 + bx + c$ has vertex P and the parabola $y = -x^2 + dx + e$ has vertex Q, where P and Q are *distinct* points. The two parabolas also intersect at P and Q.

(a) Prove that $2(e - c) = bd$.

(b) Prove that the line through P and Q has slope $\frac{1}{2}(b + d)$ and y-intercept $\frac{1}{2}(c + e)$.

END OF PROBLEM SET 1