

The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2018–2019

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel,

The London Mathematical Society and The Scottish International Education Trust.

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Senior Division: Problems 1

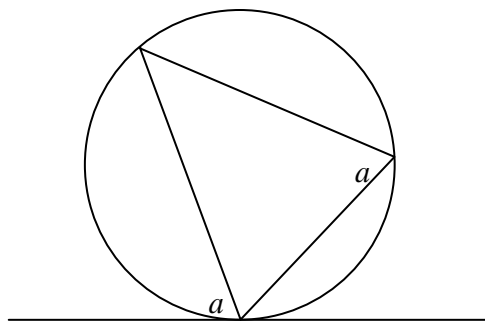
S1. Three expert logicians played a game with a set of 21 cards each with a different two-digit prime number. Each drew a card and held it up so that they could not see the number on their own card but could see the number on the cards of each of the others. Ali, Bobby and Charlie in turn were then asked two questions, namely "Is your number the smallest of the three?" and "Is your number the largest of the three?". In the first round all three answered "Don't know" to both questions. The same happened in rounds two and three. In round 4 Ali answered "Don't know" to the first question. What did Ali answer to the second question and what numbers did Bobby and Charlie have?

S2. Consider a square with side 15 cm and an equilateral triangle with the same perimeter. Which has the greater area? And by how much?

S3. (i) Prove the alternate segment theorem, which states that the angle between the tangent and chord at the point of contact is equal to the angle in the alternate segment.

(ii) Two circles touch internally at M . A straight line touches the inner circle at P and cuts the outer circle at Q and R .

Prove that $\angle QMP = \angle RMP$.



S4. Find all the positive integers k for which $7 \times 2^k + 1$ is a perfect square.

S5. My husband and I recently attended a party at which there were four other married couples. No one shook hands either with themselves or with their spouse and no one shook hands with the same person more than once. After all the handshakes were over, I asked each person, including my husband, how many hands they had shaken. To my surprise each gave a different answer. How many hands did my husband shake?

END OF PROBLEM SET 1