The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2017-2018

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel, The London Mathematical Society and The Scottish International Education Trust.

The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

Particular thanks are due to the Universities of Aberdeen, Edinburgh, Glasgow, Heriot Watt, St Andrews, Stirling, Strathclyde and to Gryffe Academy, Kelvinside Academy and Northfield Academy.

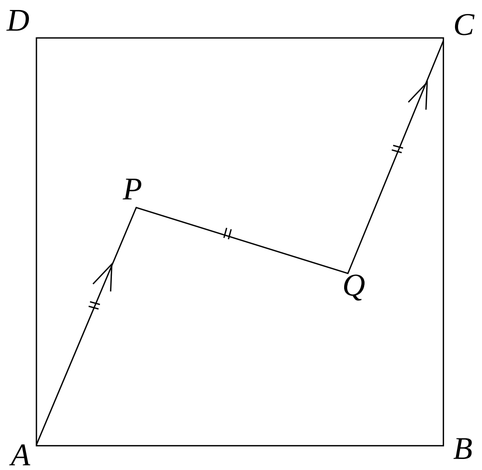
Senior Division: Problems 2

S1. Goliath and David play a game in which there are no ties. Each player is equally likely to win each game. The first player to win 4 games becomes the champion, and no further games are played. Goliath wins the first two games. What is the probability that David becomes the champion?

S2. Let n be a three-digit number and let m be the number obtained by reversing the order of the digits in n. Suppose that m does not equal n and that $n + m$ and $n - m$ are both divisible by 7. Find all such pairs n and m.

S3. $ABCD$ is a square. Points P and Q lie within the square such that AP, PQ and QC are all the same length and AP is parallel to QC.

Determine the minimum possible size of $\angle DAP$.



S4. Determine all values of x for which

$$\left(\sqrt{x}\right)^{\log_{10} x} = 100.$$

S5. In a quadrilateral $PQRS$, the sides PQ and SR are parallel, and the diagonal QS bisects angle PQR. Let X be the point of intersection of the diagonals PR and QS.

Prove that $\dfrac{PX}{XR} = \dfrac{PQ}{QR}$.

In a triangle ABC the lengths of all three sides are positive integers. The point M lies on the side BC so that AM is the internal bisector of the angle BAC. Also, $BM = 2$ and $MC = 3$.

What are the possible lengths of the sides of the triangle ABC?

END OF PROBLEM SET 2