The Scottish Mathematical Council

MATHEMATICAL CHALLENGE 2017-2018

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

Senior Division: Problems 1

S1. In a trapezium *PQRS*, *PQ* is parallel to *SR* and $\angle SPQ = \angle RQP = 135°$. The trapezium contains an inscribed circle and the length of *PQ* is 1 cm. What is the **exact** length of *QR*?

S2. Each of the six faces of a solid cube is divided into four squares as indicated in the diagram. Starting from vertex *P* paths can be travelled to vertex *Q* along connected line segments. Each movement along a path must take one closer to *Q*. How many possible paths are there from *P* to *Q*?
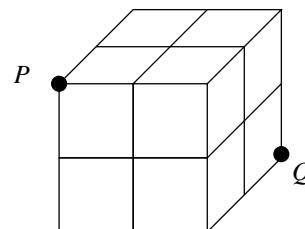
S3. In a magic square, the sum of the numbers in each diagonal, row and column is the same. What is the value of *y* + *z* in this 3 × 3 magic square?

v	24	*w*
18	*x*	*y*
25	*z*	21

S4.
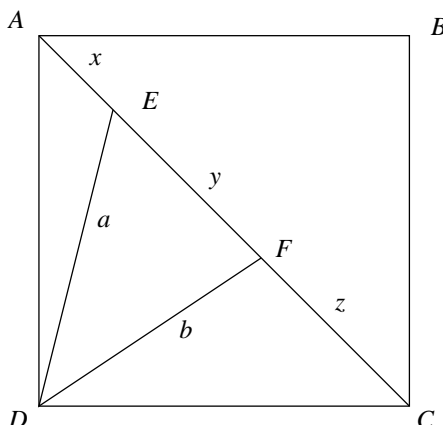
In the diagram, *ABCD* is a square. Points *E* and *F* are chosen on *AC* so that angle *EDF* is 45°. If *AE* = *x*, *EF* = *y* and *FC* = *z*, prove that

$$y^2 = x^2 + z^2$$

S5. (a) A class of 15 is to be divided into groups of three for practical work. There is one pair of twins in the class. Show that if the groups are selected at random the probability that the twins are in the same group of three is $\dfrac{1}{7}$.

(b) If there were two pairs of twins in the class determine the probability that there would be at least one group containing a pair of twins.

END OF PROBLEM SET 1