The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2016−2017

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel, The London Mathematical Society and The Scottish International Education Trust.

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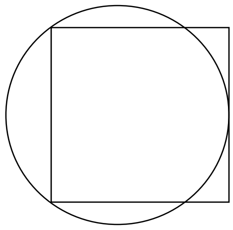
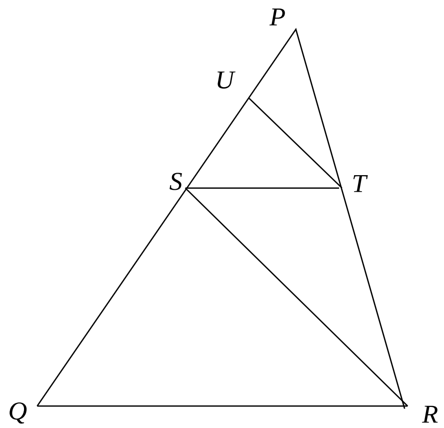
Senior Division: Problems 2

S1. In the diagram the square has two of its vertices on the circle of radius 1 unit and the other two vertices lie on a tangent to the circle. Find the area of the square.



S2.



In the diagram ST is parallel to QR, UT is parallel to SR, $PU = 4$ cm and $US = 6$ cm. Find the length of SQ.

S3. Show that the equation of any circle passing through the points of intersection of the ellipse $(x + 2)^2 + 2y^2 = 18$ and the ellipse $9(x - 1)^2 + 16y^2 = 25$ can be written in the form

$$x^2 - 2ax + y^2 = 5 - 4a$$

where a is a constant.

S4. There are 6 pairs of siblings from 6 different families.

(a) How many ways can they be divided into *two* teams so that both siblings of a pair are never in the same team?

(b) How many ways can they be divided into *three* teams so that both siblings of a pair are never in the same team?

S5. Two forgetful friends agree to meet in a coffee shop one afternoon but each has forgotten the agreed time. Each remembers that the agreed time was somewhere between 2 pm and 5 pm. Each decides to go to the coffee shop at a random time between 2 pm and 5 pm, wait for half an hour, and leave if the other doesn't arrive. Find the probability that they meet.

END OF PROBLEM SET 2