The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2015-2016

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel, The London Mathematical Society and The Scottish International Education Trust.

The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

Particular thanks are due to the Universities of Aberdeen, Edinburgh, Glasgow, St Andrews, Stirling, Strathclyde and to Bearsden Academy, Kelvinside Academy and Northfield Academy.

Senior Division: Problems 2

S1. A radio ham places an aerial mast where it gives the best reception on the roof of his rectangular garage. He then fixes wire supports from the top of the mast to each corner of the roof. The lengths of two opposite supports are seven metres and four metres and the length of one of the others is 1 metre. Find the length of the remaining support.

S2. A large board has 1000 + signs and 999 − signs written on it. Any two symbols can be deleted provided they are replaced as follows:

- if the deleted symbols are the same, they are replaced by a +
- if the deleted symbols are different they are replaced by a −.

Repeat this process until there is only one symbol left. Which symbol is it and why?

S3. There are five beads on a metal ring, each with a number on. When the beads are numbered 1,2,3,4,5 consecutively round the ring, show that it is possible to write every whole number from 1 to 15 as the total of the numbers on adjacent beads.

Now assume that you are allowed to number the beads yourself. Each bead may be labelled with *any* positive whole number, but the numbers do not have to be in sequence and they do not have to form a set of five consecutive numbers. What is the highest number N for which you can number the beads in such a way that you can write every whole number from 1 to N as the total of the numbers on adjacent beads?

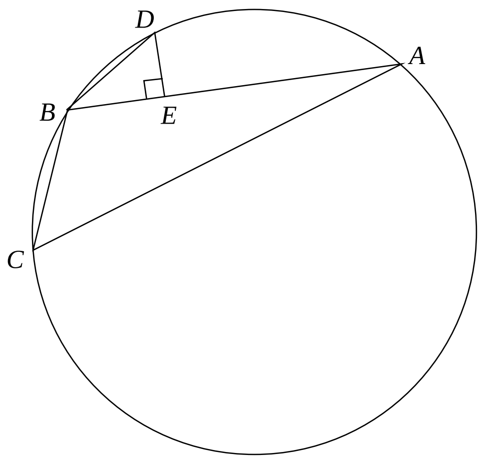
S4. I have five regular tetrahedra that I use as dice. On each face of each tetrahedron is one of the numbers 1, 2, 3 or 4 (where numbers may be repeated on faces of a single tetrahedron). No two tetrahedra use the same set of four numbers, and for each tetrahedron the sum of the four numbers is 10.

I play a game with my friend using four of these dice. He picks up a die, then I pick up a die, and we both throw. Whoever has the higher face-down number wins, or it may be a draw.

You would expect that this game to be fair, since the total on each of the five dice is 10. However by carefully selecting which one of the five dice to leave out before we start to play, I can ensure that, whichever die my friend chooses, I have a better chance of winning than he does. Which die must I leave out?

We then play the game with the remaining four dice (after leaving out the die selected in the previous paragraph). If my friend chooses the die with faces numbered 2, 2, 2, 4, which die will give me the highest probability of winning, and with what probability?

S5. In the figure, D is the midpoint of the arc AC of a circle. The point B is on the arc between C and D, and E is the foot of the perpendicular from D to AB. Show that E is the midpoint of the path from A to C along the line segments AB and BC.



END OF PROBLEM SET 2