## MATHEMATICAL CHALLENGE 2015-2016

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions. CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel, The London Mathematical Society and The Scottish International Education Trust.
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## Senior Division: Problems 1

S1. I have two blue dice and one red die.
I use the blue dice to play a simple game: if I roll a double six, I win. Otherwise, I lose.
I also roll the red die. If I roll a one, I'll lie about whether I've won or lost the game; if I roll any other number, I'll tell the truth.

I roll all three dice.
I turn to you and say "I won!".
What is the probability that I did in fact win the game?
S2. A coach travels over a hilly route from town A in the highlands to town B by the coast. Going uphill it travels at 42 mph , going downhill it travels at 56 mph and on level ground it travels at 48 mph . It takes 2 hours and 20 minutes to travel from A to B and 2 hours and 40 minutes to travel back. Find the distance between A and B.

S3. A convex polygon with 12 sides is inscribed in a circle. This polygon has six sides of length $\sqrt{2}$ and six sides of length $\sqrt{24}$ in some order. What is the radius of the circle?

S4. The diagram shows a tetrahedron $P R W U$ which fits snugly inside a cube PQRSTUVW.
Find the ratio of the surface area of the cube to the surface area of the tetrahedron .


S5. Let $n$ be a three-digit number and let $m$ be the number obtained by reversing the order of the digits in $n$. Suppose that $m$ does not equal $n$ and that $n+m$ and $n-m$ are both divisible by 7. Find all such pairs $n$ and $m$.

