The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2014−2015

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel, The London Mathematical Society and The Scottish International Education Trust.

The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

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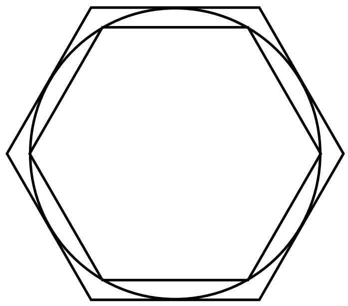
Senior Division: Problems 2

S1. A fruit drink manufacturer has a mixture of 100 litres containing w% of pure orange juice. By adding x litres of a mixture containing y% of pure orange juice he wishes to produce a mixture containing z% of pure orange juice. Find the value of x in terms of w, y and z.

S2. A regular hexagon circumscribes a circle which circumscribes another regular hexagon.

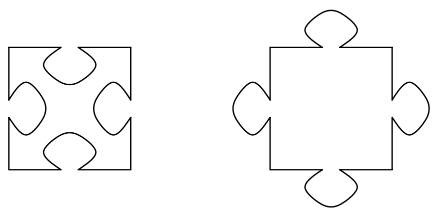
The inner hexagon has an area of 3 square units. What is the area of the outer hexagon?



S3. Four pirates were shipwrecked on a remote island. When they explored the island they found a cache of gold coins and agreed to share their haul equally. The night before they were due to divide it up, one pirate, worried that he would not get his fair share, secretly noticed that if he removed one coin, then the remainder would be exactly divisible by 4. So he took a coin and took a quarter of the remainder. Soon after, a second pirate decided to make sure of his share. Again, he found that, if he took one coin from what was left by the first pirate, the remainder would be divisible by 4. So he took a coin and took a quarter of what was left. The same thing was done in turn by the third and fourth pirates.

When they gathered the next morning to divide up the cache, with each pirate looking as innocent as possible, they found that the number of coins left was divisible by 4. What is the smallest number of coins in the original cache for this to be possible?

S4. A simple jigsaw puzzle has 225 pieces and is made from a 15×15 board where each square has area 1. All the internal (i.e. non-edge) pieces are of one of the two types shown and these types alternate along each row and column. If the ratio of the area of all the edge-pieces together to the area of the whole puzzle is $\frac{169}{675}$, how many pieces of each type are there? What is the ratio of the area of the two pieces shown?



S5. The court mathematician once received his salary for a year's services all at one time and all in silver pound coins. He proceeded to arrange his coins in nine unequal piles so that in a 3×3 array, the number in each pile formed a magic square. The king looked and admired but complained that there was not a single prime number in this magic square. "If I had but nine coins more," said the mathematician, "I could add one coin to each pile and make a magic square with every number prime". The king was about to give him nine more coins when the court jester said "Wait". Then he removed one coin from each pile and this also gave a magic square with every number prime. What is the smallest number of coins that the mathematician could have received for his year's salary?

END OF PROBLEM SET 2

CLOSING DATE FOR RECEIPT OF SOLUTIONS : 13 February 2015

Look on the SMC web site: www.scot-maths.co.uk for information about Mathematical Challenge