The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2014−2015

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel, The London Mathematical Society and The Scottish International Education Trust.

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Senior Division: Problems 1

S1. Two ferry boats set out at the same time from opposite banks of a loch. One boat is faster than the other and they pass each other at a point 650 metres from the nearer bank. After arriving at their destinations, each boat remains for 15 minutes to change passengers and then sets out on the return journey. This time, they meet at a point 350 metres from the other bank. How wide was the loch?

S2. Given that $x^2 + \frac{1}{x^2} = 7$ and $x > 0$, find the exact value of $x^5 + \frac{1}{x^5}$ in its simplest form.

S3. The lengths of the diagonals of a rhombus are 6 and 8. An equilateral triangle inscribed in this rhombus has one vertex at an end-point of the shorter diagonal and one side parallel to the longer diagonal. Determine the length of a side of this triangle.
Express your answer in the form $k(4\sqrt{3} - 3)$ where k is a vulgar fraction.

S4. In the following multiplication, each letter represents a different digit between 1 and 9.

$$\begin{array}{cccc} & A & B & C \\ \times & & D & E \\ \hline & F & E & C \\ D & E & C & \\ \hline H & G & B & C \end{array}$$

What two numbers are being multiplied together?

S5. A *multiply-perfect number* is one for which the sum of its distinct factors, including 1 and the number itself, is an integer multiple of the given number. Show that 30240 is a multiply-perfect number.

END OF PROBLEM SET 1