The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2013−2014

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel, The London Mathematical Society and The Scottish International Education Trust.

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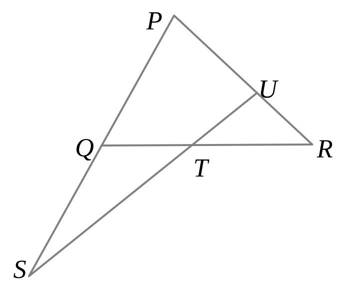
Senior Division: Problems 2

S1. PQR is any triangle. The side PQ is extended to S where $PQ = QS$. The point U divides the side PR in the ratio $3 : 2$. The point T is where the lines QR and SU cross.

Find the ratio $\dfrac{QT}{QR}$.

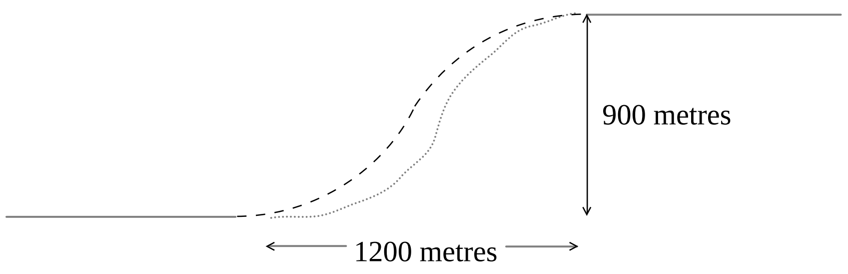


S2. Dots are arranged in a rectangular grid with 4 rows and n columns. Consider different ways of colouring the dots, in which each dot either red or blue. A colouring is 'good' if no four dots of the same colour form a rectangle with horizontal and vertical sides.

Find the maximum value of n for which there is a good colouring.

S3. A triangle has sides of length 4, 7 and 9 units. Find the length of the longest median. **Show your reasoning.**

S4.



Two straight sections of a road, each running from east to west, and located as shown, are to be joined smoothly by a new roadway consisting of arcs of two circles of equal radius. The existing roads are to be tangents at the joins and the arcs themselves are to have a common tangent where they meet. ′Find the length of the radius of these arcs.

S5. Let

$$f(x) = x^n + a_1 x^{n-1} + \ \dots \ + a_n,$$

where $a_1, a_2, \dots, a_n$ are given numbers. It is given that $f(x)$ can be written in the form

$$f(x) = (x + k_1)(x + k_2)\dots(x + k_n).$$

By considering $f(0)$, or otherwise, show that $k_1 k_2 \dots k_n = a_n$.

Show also that

$$(k_1 + 1)(k_2 + 1)\dots(k_n + 1) = 1 + a_1 + a_2 + \ \dots \ + a_n$$

and give the corresponding result for $(k_1 - 1)(k_2 - 1)\dots(k_n - 1)$.

Hence, find the roots of the equation

$$x^4 + 22x^3 + 172x^2 + 552x + 576 = 0,$$

given that they are all integers.

END OF PROBLEM SET 2