The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2013−2014

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel, The London Mathematical Society and The Scottish International Education Trust.

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Senior Division: Problems 1

S1. On a tiny remote island where the death sentence still exists a man can be granted mercy after receiving the death sentence in the following way:

- he is given 18 white balls and 6 black balls.
- he must divide them between three boxes with at least one ball in each box.
- he is then blindfolded and must choose a box at random and then a single ball from within this box.

He receives mercy only if the chosen ball is white.

Find the probability that he receives mercy when he distributes the balls in the most favourable manner.

S2. Three types of item, A, B and C, are for sale. Items of type A sell at 8 for £1. Items of type B sell for £1 each. Items of type C sell for £10 each. A selection of 100 items which includes at least one of each type costs £100. How many items of type B are there in the selection?

S3. Prove that the area of a regular hexagon with side of length a is $\frac{3\sqrt{3}}{2}a^2$.

Not all equiangular hexagons are regular: find a formula for the area of an equiangular hexagon with sides of length a, b, a, b, a, b in that order.

S4. Naismith's rule enables hillwalkers to estimate route times according to the following rule (updated from its original formulation in 1892): allow one hour for every 5 km measured on the map, plus one hour for every 600 m of ascent.

A group of hillwalkers was confronted by an exactly conical hill with diameter 4 km rising from a plain. Some of the group chose to climb to the summit of the hill, whereas the rest of the group chose to walk on the flat round its circular base. Neither group stopped en route, and the summit group arrived at the meeting point on the opposite side of the hill 20 minutes after the walk round group.

Assuming Naismith's rule is correct, determine the height of the hill above the plain. Give your answer to the nearest metre.

S5. A cube of edge 6 cm is divided into 216 unit cubes by planes parallel to the faces of the cube. A sphere of diameter 6 cm sits inside the cube so that the faces of the cube are tangents to the sphere. How many complete unit cubes are contained within this sphere?

END OF PROBLEM SET 1