The Scottish Mathematical Council
www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2012–2013

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

Senior Division: Problems 2

S1. Show that the maximum range of an aeroplane is extended by a factor of $\frac{1}{3}$ when there is a second identical support plane which sets off at the same time to provide in-air refuelling. The support plane must return safely to the starting point.

Now consider the situation where there are two identical support planes which can (instantaneously) refuel each other or the original plane as required. The support planes set off at the same time as the original plane and both must return safely to the starting point. By how much can the maximum range of the plane be extended?

S2. In the diagram angle *PRQ* is a right angle and *PS* and *SR* are both length 1 cm. *QR* is length 2cm. Find the exact value of $\tan \theta$.
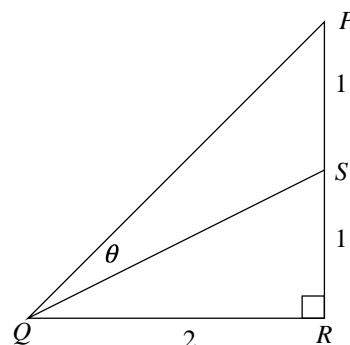
S3. Four apparently identical small objects have weights *a*, *b*, *c* and *d* such that
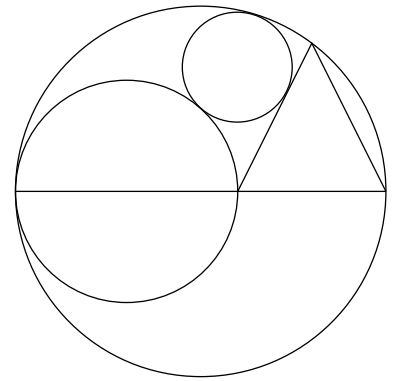$$a < b < c < d.$$
It is also known that
$$a > \frac{2}{3}b$$
and
$$c < \frac{3}{4}d.$$
Using only a balance, show how it is possible to pick out the heaviest object in just 2 weighings.

S4. I was reading a novel aimed at children and noticed that exactly half of the page numbers began with '1'. Work out the number of pages in the novel.

S5. Instructions for drawing this diagram are as follows:

Split the diameter of the large circle into two parts. On one part, draw a second circle with that part as diameter. On the other part, draw an isosceles triangle with that part as base and the other vertex on the circumference of the large circle. Now draw a third circle so that it touches the other two circles and the triangle, as shown in the diagram.

Show that the centre of the third circle must lie on the line perpendicular to the diameter of the large circle and meeting this diameter at the point where the second circle and triangle touch.

END OF PROBLEM SET 2