

The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2012–2013

Entries must be the unaided efforts of individual pupils.

Solutions must include explanations and answers without explanation will be given no credit.

Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

*The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel,
The London Mathematical Society and The Scottish International Education Trust.*

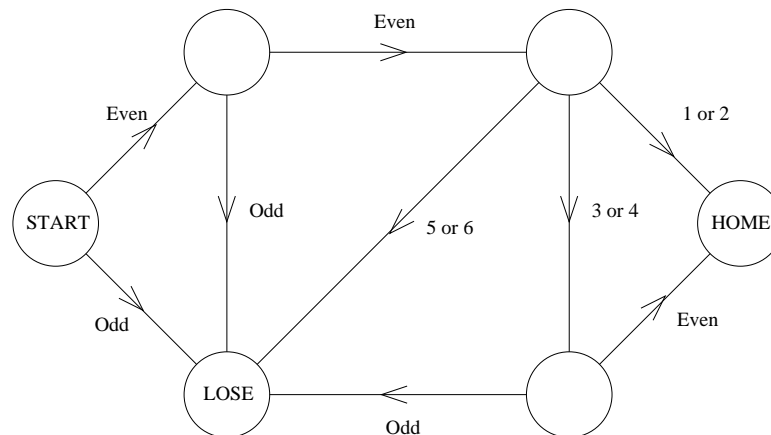
The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

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Senior Division: Problems 1

- S1.** Four spheres each of radius 10 cm lie on a horizontal table so that the centres of the spheres form a square of side 20 cm. A fifth sphere of radius 10 cm is placed on them so that it touches each of the spheres without disturbing them. How far above the table is the centre of the fifth sphere?

S2.



A counter placed on the start circle moves in the direction determined by the throw of a normal six-sided die. What is the probability of reaching the HOME circle?

- S3.** Crispin had been learning about perfect squares. He had then taken some of the number blocks borrowed from his little sister Suzy and arranged them to form five numbers, exactly one of which was a perfect square. He then went off to fetch his father to see if he could work out which of the numbers he had formed was the perfect square.

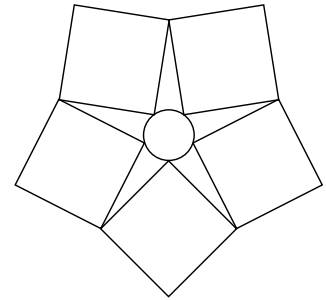
When they returned, however, they discovered that Suzy had been playing with the number blocks again. She had rearranged the blocks in the first number and also in the second number. In addition, she had taken away all but the last two of the blocks in the third number, all but the last block in the fourth number and all the blocks in the fifth number. This is what the remaining blocks looked like:

- (1) 3 5 5 8 7 3 2
- (2) 7 3 7 4 3 7 3
- (3) 7 5
- (4) 8
- (5)

Crispin's father looked at the blocks and said, "Never mind, I can still tell you which number was the perfect square". Can you?

- S4.** Consider the sequence of all positive integers for which the sum of the digits is divisible by 7, arranged in order of increasing magnitude.
- (a) Write down the first members of this sequence less than 100.
 - (b) What is the maximum difference between consecutive members of the whole sequence? Justify your answer.

- S5.** In the diagram, five identical squares are arranged symmetrically round a circle so that their vertices touch each other and one vertex of each square lies on the circle. Starting with a circle with radius equal to the length of a side of the squares, how many squares would it take to be similarly arranged?



END OF PROBLEM SET 1