The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2011−2012

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, Professor L E Fraenkel,
The London Mathematical Society and The Scottish International Education Trust.
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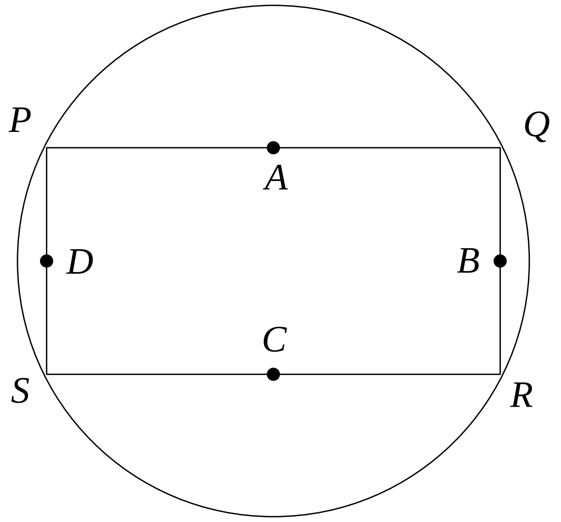
Senior Division: Problems 2

S1. Andy is standing at a bus stop near his house. Through a small window, he can see the reflection of a television in a large mirror. The television set is mounted on the same wall of the house as the window and the mirror is on the opposite wall. He also notices that the reflection he sees through the small window is the full width of the TV but no more.

He wonders how wide his neighbour's TV is. But as the house is exactly like his own he can work it out. The small window is 50 cm wide and the room is 4 m deep. Furthermore he is exactly 10m from the nearest point on the front wall of the house on which the window and the TV are. How does he calulate this and what is the width of the TV?

S2. An employee submitted a claim for travelling expenses. The accountant dealing with the claim made a mistake in which he interchanged the numbers of pounds and pence. This resulted in a greater payment being made than was due. Not noticing the error, the employee spent £3.19 on a magazine from the payment on his way home. When he checked his cash, he found that the sum he now had was exactly 7/5 of his original claim. How much did he claim for travelling expenses?

S3. The vertices of a rectangle *PQRS* lie on a circle of radius r. The points *A*, *B*, *C* and *D* are the midpoints of *PQ*, *QR*, *RS* and *SP* respectively. Determine, with proof, the greatest possible area of the quadrilateral *ABCD*.



S4. Determine the equation of the circle which satisfies these conditions:

- the graph is tangential to the x-axis;
- it passes through $(-3, 10)$ and $(0, 1)$;
- the two points where the graph intersects the positive y-axis are 8 units apart.

S5. In how many ways can the number 1 000 000 be expressed as the product of three positive integers a, b, c where $a \leqslant b \leqslant c$?

END OF PROBLEM SET 2