The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2011−2012

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, Professor L E Fraenkel,
The London Mathematical Society and The Scottish International Education Trust.
The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.
Particular thanks are due to the Universities of Aberdeen, Edinburgh, Glasgow, Heriot Watt, Stirling, Strathclyde, and to Preston Lodge High School, Bearsden Academy, Beaconhurst School and Northfield Academy.

Senior Division: Problems 1

S1. The rectangular floor of a room is completely tiled with whole tiles, each of which is 15 cm square. The black tiles form a border of width one tile round the room. Within the black border all the tiles are red. There are exactly twice as many red tiles as black ones. Determine the possible lengths and corresponding widths for the room.

S2. Show that there is only one set of different positive integers, x, y, z, such that

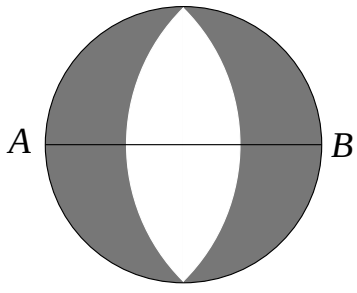
$$1 = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

i.e. 1 can be expressed as the sum of the reciprocals of three different positive integers in only one way.

Deduce that, if n is any odd integer greater than 3, then 1 can be expressed as the sum of n reciprocals of different positive integers.

For which even integers is this possible? **Justify your answer**.

S3. A circle has radius 1cm and AB is a diameter. Two circular arcs of equal radius are drawn with centres A and B. These arcs meet on the circle as shown. Calculate the shaded area.



S4. The thinking power of a multi-headed dragon depends on how many heads it has. The thinking power of a 'weyr' of dragons is the product of the number of heads on the individual dragons. A particular weyr has 100 heads available, how many dragons with what number(s) of heads will maximise the thinking power of the group?

S5. Show that the ratio of the area of a regular n-gon inscribed in a circle to the area of a regular n-gon circumscribing the same circle is $\cos^2\frac{\pi}{n}$: 1.