The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2010−2011

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, Professor L E Fraenkel,
The London Mathematical Society and The Scottish International Education Trust.

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Senior Division: Problems 1

S1. Katie had a collection of red, green and blue beads. She noticed that the number of beads of each colour was a prime number and that the numbers were all different. She also observed that if she multiplied the number of red beads by the total number of red and green beads she obtained a number exactly 120 greater than the number of blue beads. How many beads of each colour did she have?

S2. Ant and Dec had a race up a hill and back down by the same route. It was 3 miles from the start to the top of the hill. Ant got there first but was so exhausted that he had to rest for 15 minutes. While he was resting, Dec arrived and went straight back down again. Ant eventually passed Dec on the way down just half a mile before the finish.

Both ran at a steady speed uphill and downhill and, for both of them, their downhill speed was one and a half times faster than their uphill speed. Ant had bet Dec that he would beat him by at least a minute.

Did Ant win his bet?

Explain your answer.

S3. Two numbers contain the same digits in a different order. Explain why the difference between the numbers is always a multiple of 9.

S4. Let ABC be an acute-angled triangle with sides of lengths a, b, c and area X. Show that the radius of the circle through A, B and C is $\dfrac{abc}{4X}$.

S5. John said "I am told that there is only one number between 2 and 200 000 000 000 000 which is a perfect square, a perfect cube and also a perfect fifth power. I am sure there must be more than one, but I have looked at all the numbers up to 100 000 and I haven't found any! I am getting fed up doing this."

END OF PROBLEM SET 1