The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2008−2009

Entries must be the unaided efforts of individual pupils. Solutions must include explanations.

Answers without explanation will be given no credit.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, Professor L E Fraenkel,

The London Mathematical Society and The Scottish International Education Trust.

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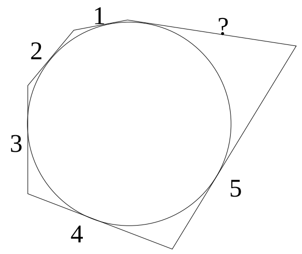
Senior Division: Problems 2

S1. In the hexagon shown, all edges are tangent to the circle. If their lengths are 1, 2, 3, 4 and 5 as illustrated, what is the length of the remaining edge?



S2. An unusual team marathon is being held. Each team has two members who run and cycle over the course which is 42 km long. The rules are:

1. Each team starts together at the start line.
2. Each team is allowed only one bike and only one team member may ride the bike at any time. (This usually means that one member cycles at the start, puts the bike down at some point and then runs to the finish line. The other team member starts by running. When he or she reaches the bike, they pick it up and cycle to the finish line).
3. Both team members have to cross the finish line.
4. The time recorded for a team is the time for the second member to cross the finish line or the time of both members if they cross the finish line together. (This means that teams should work out before the race starts exactly where they should leave the bike to give the fastest recorded time).

Team	Member 1	Running	Cycling	Member 2	Running	Cycling
A	Zak	12	28	Sandra	12	28
B	Archie	16	35	Kirsty	10	15
C	Liam	14	25	Niamh	10	35

The table shows the steady running and cycling speeds, all in km/hr, for the three teams. Determine for each team, their best possible time for the race.

S3. Every polyhedron satisfies Euler's formula. This states that, if V is the number of vertices of the polyhedron, E the number of edges and F the number of faces, then $V - E + F = 2$.

A *rhombic triacontahedron* is a semi-regular polyhedron which has 30 congruent faces which are rhombuses and two types of vertices. At vertices of Type 1, three rhombuses meet and the angles there are all obtuse; at vertices of Type 2, five rhombuses meet and the angles are all acute. Use Euler's formula to help determine how many edges there are and how many vertices of each Type.

A new semi-regular polyhedron is formed by suitably truncating all the vertices of Type 2 of the rhombic triacontahedron described above. In this case all the vertices have the same number of edges meeting at them but there are two Types of faces. Describe this new polyhedron, giving the Types of faces, how many of each type there are and how many edges and vertices.

S4. Consider a rectangular grid of points with r rows and c columns. The distances between rows and between columns are exactly the same, and can be taken as one unit.

(a) Show that the area of the complete grid of points can be determined from $\frac{1}{2}E + I - 1$ where E is the number of points on the sides of the grid and I is the number of points within the grid.

(b) Now connect two opposite corner points of the rectangle, and consider one of the two right angled triangular regions. Show that exactly the same result holds for the area of one of these triangular regions, i.e. the area of the triangle is given by $\frac{1}{2}E + I - 1$ where E is the number of points on the sides of the triangle and I is the number of points within the triangle.

(c) Finally consider the triangular region formed when the two end points on one side of the rectangular grid are connected to one of the interior points of the opposite side. Again show that the area of the triangle is given by $\frac{1}{2}E + I - 1$ where E is the number of points on the sides of the triangle and I is the number of points within the triangle.

S5. In a Fantasy Football League, at a certain stage in the season, Hearts had lost some games but had won 8 games and drawn 9. Having played exactly the same number of games, Aberdeen had an average of 1.07 points per game. Which team was ahead at this stage? In this year, each team was playing a total of 36 league games in a season, there are 3 points for a win and 1 for a draw and the average number of points scored by Aberdeen is given correct to two decimal places.

END OF PROBLEM SET 2