The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2008−2009

Entries must be the unaided efforts of individual pupils. Solutions must include explanations.

Answers without explanation will be given no credit.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, Professor L E Fraenkel,

The London Mathematical Society and The Scottish International Education Trust.

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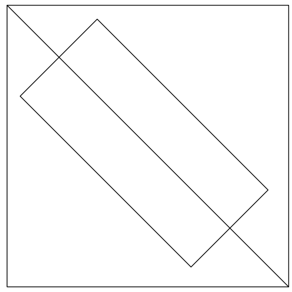
Senior Division: Problems 1

S1. Show that if a rectangle, which is twice as long as it is broad, can fit diagonally into a square as shown below, then it can also fit into the square with its sides parallel to the sides of the square.

Is this true if the rectangle is three times as long as it is broad? **Explain your answer.**



S2. A queue of slow moving traffic is 5 miles long. It takes 15 minutes to pass a particular road sign. A police car takes a total of 20 minutes driving at constant speed to travel from the back of the queue to the front and to return to the back. How fast does the police car travel?

S3. In 1946, an American numerologist, Professor Humbug, predicted the downfall of the USA in the year 2141 based on what he called a profound mathematical discovery depending on the following expression:

$$1492^n - 1770^n - 1863^n + 2141^n$$

He spent many months calculating the value of this for $n = 1, 2, 3$ and so on up to 1945 and found the remarkable fact that the result was always divisible by 1946. Since the years 1492, 1770 and 1863 are all important in American history, he claimed that 2141 would also be significant – hence his prediction.

Show how he could have saved himself months of work.

Hint: $(x^n - y^n) = (x - y)(x^{n-1} + \ \dots \ + y^{n-1})$

S4. A circle of radius 15 cm intersects another circle of radius 20 cm at right angles.

What is the difference of the areas of the non-overlapping portions?

What is the sum of the areas of the non-overlapping portions?

S5. Show that the square of any integer leaves a remainder of 0, 1, 4 or 7 when divided by 9.

Use this to establish the following condition that a number which is a perfect square must satisfy the following:

> For a number that is a perfect square, add up its digits to form a second number. If that number has more than one digit, add up its digits to form a third number. Continue until you obtain a single digit number. That final number must be 1,4, 7 or 9.

END OF PROBLEM SET 1