The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2006−2007

Entries must be the unaided efforts of individual pupils. Solutions must include explanations.

Answers without explanation will be given no credit.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, Professor L E Fraenkel,

The London Mathematical Society and The Scottish International Education Trust.

The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.

Particular thanks are due to the Universities of Aberdeen, Dundee, Edinburgh, Glasgow, Heriot-Watt, Paisley, Stirling, Strathclyde, and to Preston Lodge High School, Bearsden Academy, and Turriff Academy.

Senior Division: Problems 2

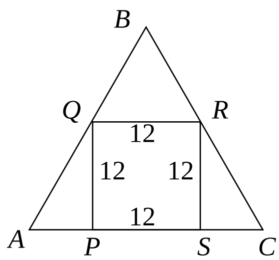
S1. Daryl put some black stones and some white stones into a bag. He then asked Ran to reach into the bag, without looking in, and draw out a stone. Ran drew out a black stone. Daryl asked Ran to draw out another stone and, once again, Ran drew out a black stone.

"There must be more black than white stones in the bag ," said Ran. "I wonder what the probability is of my drawing a black stone on a third try?"

Daryl replied," Exactly nine tenths of what it was of drawing a black stone on your first draw."

Daryl told Ran that he had put "ten, give or take two or three" stones into the bag. How many stones were in the bag at the start?

S2. *PQRS* is a square of side 12 cm which fits inside an equilateral triangle *ABC* in such a way that each of *P*, *Q*, *R* and *S* is on a side of *ABC* as shown. Calculate the area of triangle *ABC*.



S3. Let $x_1, x_2, \ldots, x_{100}$, denote 100 positive numbers whose sum is 100.

(i) Explain why there must be at least one suffix i such that $x_i \leqslant 1$.

(ii) Explain why there must be at least one suffix i such that $x_i + x_{i+1} \leqslant 2$.

(iii) Find 100 such positive numbers, all different, such that $x_i + x_{i+1} \leqslant 2$ for all i.

S4. A dishonest barman removes three litres of wine from a barrel, replacing them with water and mixing the contents. He repeats the theft twice more, removing in total 9 litres and replacing them with water. As a result the wine remaining in the barrel is half its former strength. How much wine did the barrel originally hold? (Your answer may given to two decimal places.)

S5. Find the largest integer which is both an n-digit number and an nth power.

END OF PROBLEM SET 2