The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2006−2007

Entries must be the unaided efforts of individual pupils. Solutions must include explanations.

Answers without explanation will be given no credit.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, Professor L E Fraenkel,

The London Mathematical Society and The Scottish International Education Trust.

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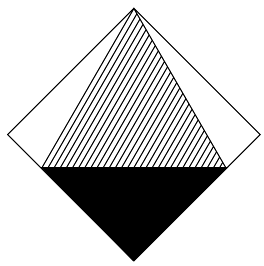
Senior Division: Problems 1

S1. In a snowball ‘fight’, where snowballs are identical spheres, your opponents have stacked their snowballs in a square pyramid. You are about to count the snowballs along the bottom edge of the opponent's stack when one appears with another snowball. After giving him a telling off, the opposition's leader takes apart the square pyramid and builds a new, triangular pyramid using all the original snowballs and the extra one. Find two possible values for the number of snowballs that your opponents now have.

S2. Three cyclists are out for the day. Two are on a tandem and one on an ordinary cycle. Disaster struck when the ordinary cycle was stolen while they were having lunch in a café. They were left with the tandem and 20 miles to go. The tandem has to have two riders and the third person walks. Anne can walk a mile in 20 minutes, Sam in 30 minutes and Oscar in 40 minutes. The tandem travels at 20 miles per hour no matter which pair is riding it. What is the shortest time for all three to get home?

S3. A group of seven girls − Ally, Bev, Chi-chi, Des, Evie, Fi and Grunt – were playing a game in which the counters were beans. Whenever a girl lost a game, from her pile of beans she had to give each of the other girls as many beans as they already had. They had been playing for some time and they all had different numbers of beans. They then had a run of seven games in which each girl lost a game in turn, in the order given above. At the end of this sequence of games, amazingly, they all had the same number of beans – 128. How many did each of them have at the start of this sequence of seven games?

S4. Pat and Jo were having a holiday in Greece and visited a temple. They noticed a wall tile as shown alongside.



The shaded triangle is equilateral with one of its vertices at a one corner of the surrounding square and the others on two of the sides of the square. Pat said “I bet the area of the black triangle is the same as the combined areas of the two white triangles”. “Don't be daft!” said Jo.

Who was right and why?

S5. Let a, b, c be the lengths of the sides of a triangle. Show that

$$\frac{a}{b+c} + \frac{b}{a+c} + \frac{c}{a+b} < 2.$$

Describe the shape of a triangle for which the expression, $\frac{a}{b+c} + \frac{b}{a+c} + \frac{c}{a+b}$, is very close to 2.

END OF PROBLEM SET 1