The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2019−2020

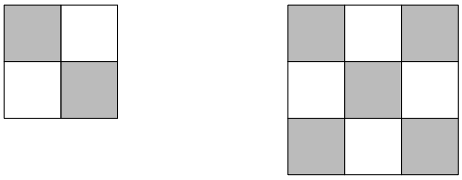
Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

Middle Division: Problems 2

M1. How many positive multiples of 7 that are less than 1,000 end with the digit 3?
How many positive multiples of 7 that are less than 10,000 end with the digits 33?

M2. A cross-country skier practices the same route from his home to his friend's home and leaves at the same time every day. He realised that when he skis at 10 mph he arrives at 4 minutes past noon, and when he skis at 15 mph he arrives at 4 minutes before noon. How fast would he have to go to reach his friend's house at noon exactly?

M3. Consider an $n \times n$ grid of squares in which the squares alternate grey and white starting with grey in the top left hand corner. A 2×2 grid and a 3×3 grid will look like:



Using a grid of a given size, two black spots are placed on two different squares chosen at random.

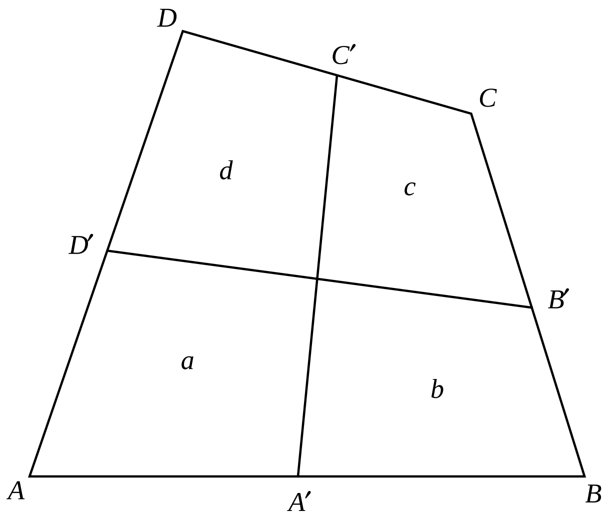
For the two grids above (i.e. $n = 2, 3$), find out in how many distinct ways this can be done. What is the probability that both spots are on grey squares?

Obtain the corresponding results for the case $n = 9$.

M4. A positive integer ends in the digit 4 and has the property that it becomes four times as large when the 4 is moved from the end and placed at the front. What is the smallest such number?

M5.

Let $ABCD$ be a quadrilateral. Let A' be the midpoint of AB, B' the mid-point of BC, C' the mid-point of CD and D' the mid-point of AD. Draw the lines $A'C'$ and $B'D'$ and let a, b, c, d be the areas of the four minor quadrilaterals as shown in the figure. Prove that $a + c = b + d$.



END OF PROBLEM SET 2