## 2016-2017 Middle Solutions 2

**M1.** Peter is walking through a train tunnel when he hears a train approaching. He knows that on this section of track trains travel at 60 mph. The tunnel has equally spaced marker posts, with post 0 at one end and post 12 at the other end. Peter is by post 7 when he hears the train. He quickly works out that whether he runs to the nearer end or the further end of the tunnel as fast as he can (at constant speed) he will just exit the tunnel before the train reaches him.
How fast can Peter run?

*Solution*

Let the length of the tunnel be $T$ miles.

As Peter is at post 7, the distances to the ends of the tunnel are $\frac{7}{12}T$ and $\frac{5}{12}T$ miles.

The train reaches the start of the tunnel (at post 12) at the time it would take Peter to run to the nearer end, i.e. the time to run a distance of $\frac{5}{12}T$ miles.

So if instead he runs to the further end (which is at post 0) he will reach $\frac{10}{12}T$ miles when the train is just entering the tunnel.

So he will have to run the remaining $\frac{2}{12}T = \frac{1}{6}T$ miles of the tunnel in the time the train takes to pass through the whole length of the tunnel at 60 mph, i.e. $\frac{T}{60}$ hours.

So Peter's speed is $\frac{\frac{T}{6}}{\frac{T}{60}} = \frac{60}{6} = 10$ mph.

**M2.** An old fashioned tram starts from the station with a certain number of men and women on board.

At the first stop, a third of the women get out and their places are taken by men. At the next stop, a third of the men get out and their places are taken by women. There are now two more women than men and as many men as there originally were women.

How many men and women were there on board at the start?

*Solution*

| | Number of men | Number of women |
|---|---|---|
| From the station | $m$ | $w$ |
| After the first stop | $m + \frac{1}{3}w$ | $\frac{2}{3}w$ |
| After the second stop | $\frac{2}{3}m + \frac{2}{9}w$ | $\frac{7}{9}w + \frac{1}{3}m$ |

There are now as many men as there were women originally:

$$\frac{2}{3}m + \frac{2}{9}w = w$$
$$\frac{2}{3}m = \frac{7}{9}w$$
$$6m = 7w$$

And there are two more women than men:

$$\frac{7}{9}w + \frac{1}{3}m = \frac{2}{3}m + \frac{2}{9}w + 2$$
$$\frac{5}{9}w = \frac{1}{3}m + 2$$
$$5w = 3m + 18$$
$$10w = 6m + 36$$
$$10w = 7w + 36$$
$$3w = 36$$
$$w = 12$$
$$m = 14$$

So there were 14 men and 12 women on the bus at the start.

**M3.** Kirsty runs three times as fast as she walks. When going to school one day she walks for twice the time she runs and the journey takes 21 minutes. The next day she follows the same route but runs for twice the time she walks. How long does she take to get to school?

*Solution*

Day 1

| | time | speed | distance |
|---|---|---|---|
| walk: | 14 mins | $v$ miles per min | speed × time = 14 $v$ miles |
| run: | 7 mins | 3v miles per min | speed × time = 21 $v$ miles |

total distance 35$v$ miles

Day 2 Let time taken be $t$ mins.

| | time | speed | distance |
|---|---|---|---|
| walk: | $t/3$ mins | $v$ miles per min | speed × time = $t/3 \,.\, v$ miles |
| run: | $2t/3$ mins | $3v$ miles per min | speed × time = $2t/3 \,.\, 3v$ miles |

total distance $7t/3.v$ miles

The distance is the same on both days and so

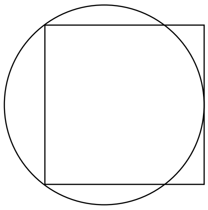
$$35v = \frac{7t}{3} \times v$$

Hence

$$t = 15$$

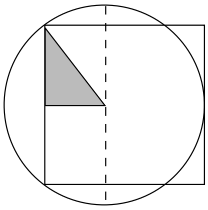
She takes 15 minutes to get to school on the next day.

**M4.**



In the diagram the square has two of its vertices on the circle of radius 1 unit and the other two vertices lie on a tangent to the circle. Find the area of the square.

*Solution*



Let the side of the square be $2x$ units.
Then the shaded right-angled triangle shown has sides $x$ and $2x - 1$ and hypotenuse 1.
By Pythagoras' theorem,

$$x^2 + (2x - 1)^2 = 1^2$$

$$x^2 + 4x^2 - 4x + 1 = 1$$

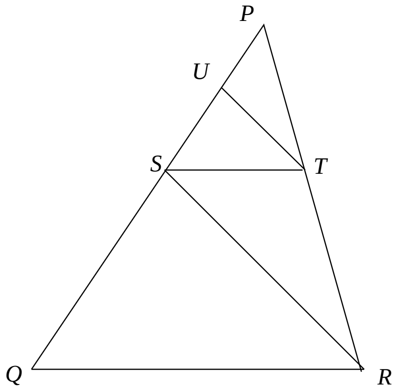
$$5x^2 - 4x = 0$$

$$x(5x - 4) = 0$$

$x = 0$ gives a degenerate triangle and square, so the required solution is $x = \frac{4}{5}$.
Hence the area of the square is

$$\left(\frac{8}{5}\right)^2 = \frac{64}{25} = 2.56 \text{ units}^2$$

**M5.**




In the diagram $ST$ is parallel to $QR$, $UT$ is parallel to $SR$, $PU = 4$ and $US = 6$. Find the length of $SQ$.

*Solution*

In triangle $PSR$ we can say

$$\frac{4}{6} = \frac{PU}{US} = \frac{PT}{TR}.$$

Now in triangle $PQR$ we can say

$$\frac{4}{6} = \frac{PT}{TR} = \frac{PS}{SQ} = \frac{10}{SQ}$$

Hence

$$SQ = 10 \times \frac{6}{4}.$$

So $SQ = 15$.