# The Scottish Mathematical Council

# MATHEMATICAL CHALLENGE 2016-2017

**Entries must be the unaided efforts of individual pupils.**

**Solutions must include explanations and answers without explanation will be given no credit.**

**Do not feel that you must hand in answers to all the questions.**

## Middle Division: Problems 1

**M1.** A victorious football team in an open-top bus is scheduled to leave the home ground and arrive at the town hall at 11 am. If the bus travels at 15 mph it will arrive 8 minutes early. However if it travels at 10 mph it will arrive 8 minutes late. At what speed must it travel to arrive at 11 am exactly?

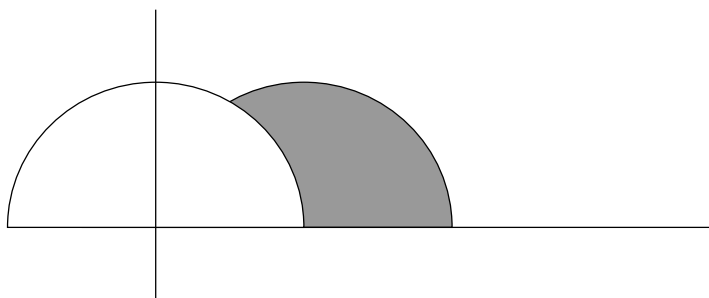**M2.** (a) Adam has a five-digit number

$$* \; * \; * \; * \; *$$

When he places a 1 at the end of this number it becomes three times larger than when he placed a 1 at the start.
Find the five-digit number.

(b) If you added a 1 in the same way to a 3-digit number how many times larger would it be?

**M3.**



The diagram shows parts of two circles, each with radius 1 unit, and centres on the $x$-axis at $x = 0$ and $x = 1$. Determine the **exact** value of the shaded area.

**M4.** A pyramid stands on horizontal ground. Its base is an equilateral triangle with sides of length $a$, the other three edges of the pyramid are of length $b$ and its volume is $V$. Show that the volume of the pyramid is

$$V = \frac{1}{12}a^2\sqrt{3b^2 - a^2}.$$

**M5.** Eight islands each have one or more air services. An air service consists of flights to and from another island, and no two services link the same pair of islands. There are 17 air services in all between the islands.
Show that it must be possible to use these air services to fly between any pair of islands.

**END OF PROBLEM SET 1**