MATHEMATICAL CHALLENGE 2013–2014

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
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Middle Division: Problems 2

M1. A block of four postage stamps, with perforations along the joins so that they can be easily separated, have values in pence as shown:

1	2
4	3

Show that it is possible to make every postage value from 1p to 10p using either a single stamp or a number of stamps joined along lines of perforations.

Using a different set of stamp values in the block of four, it is possible to make every postage value from 1p to a higher limit than 10p. Construct an arrangement of stamp values which reaches the highest possible limit.

Are there any other solutions which give this limit? Explain.

M2. A brother and sister, Peter and Fiona, are always thinking about numbers.

On his birthday Peter said "My age is a square number."

His older sister Fiona said "That's right, but the sum of our ages and the difference of our ages also give squared numbers."

Peter replied "In three years time, both our ages will be prime numbers."

Fiona replied "Three years ago, both our ages were also prime numbers."

What are the ages of Peter and Fiona now?

M3. *PQR* is any triangle. The side *PQ* is extended to *S* where
$PQ = QS$. The point *U* divides the side *PR* in the ratio $3 : 2$.
The point *T* is where the lines *QR* and *SU* cross.

Find the ratio $\dfrac{QT}{QR}$.
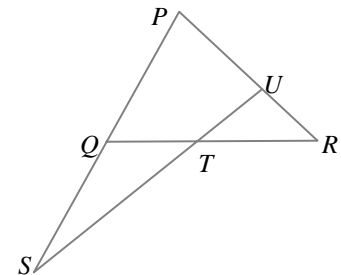
M4. Rebecca celebrated her graduation by going for a hot air balloon ride. At first the wind blew the balloon $\frac{1}{2}$ a mile due east. Then the balloon was blown $\frac{3}{4}$ of a mile southwest. Finally, it was blown $\frac{1}{2}$ a mile north and then landed. How many miles did the balloon land from the point where it was launched? Express your answer as a decimal to three places.

Do not use a scale drawing.

M5. Dots are arranged in a rectangular grid with 4 rows and *n* columns. Consider different ways of colouring the dots, in which each dot either red or blue. A colouring is 'good' if no four dots of the same colour form a rectangle with horizontal and vertical sides.

Find the maximum value of *n* for which there is a good colouring.

END OF PROBLEM SET 2