The Scottish Mathematical Council

MATHEMATICAL CHALLENGE 2012–2013

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, The Maxwell Foundation, Professor L E Fraenkel,
The London Mathematical Society and The Scottish International Education Trust.

Middle Division: Problems 1

M1. In mathematics, the notation 11! is short for $11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$. Similarly, another example is, $4! = 4 \times 3 \times 2 \times 1$ and so $4! = 24$.

What is the greatest factor of 11! that is one greater than a multiple of 6?
Note: 11! is spoken of as 'eleven factorial'.

M2. I take a certain journey and due to heavy traffic crawl along the first half of the complete distance of my journey at an average speed of 10 mph. How fast would I have to travel over the second half of the journey to bring my average speed for the whole journey up to 16 mph? And how fast to bring it up to 20mph?

M3. All dragons have a head, a body and a tail and the length of a dragon is the sum of the lengths of its head, body and tail. A mother dragon and her son were lying in the sun. They noticed that the length of the young dragon was exactly the same as the length of his mother's tail. The length of the body of the mother was three times as long as her own head which was twice as long as the head of her son. The younger dragon's body was 2 feet longer than his head and his tail was one-third of the length of the tail of his mother. When the lengths of the two dragons are added, the total is 48 feet.
Determine the lengths of the head, body and tail of each dragon.

M4. Four spheres each of radius 10 cm lie on a horizontal table so that the centres of the spheres form a square of side 20 cm. A fifth sphere of radius 10 cm is placed on them so that it touches each of the spheres without disturbing them. How far above the table is the centre of the fifth sphere?

M5.
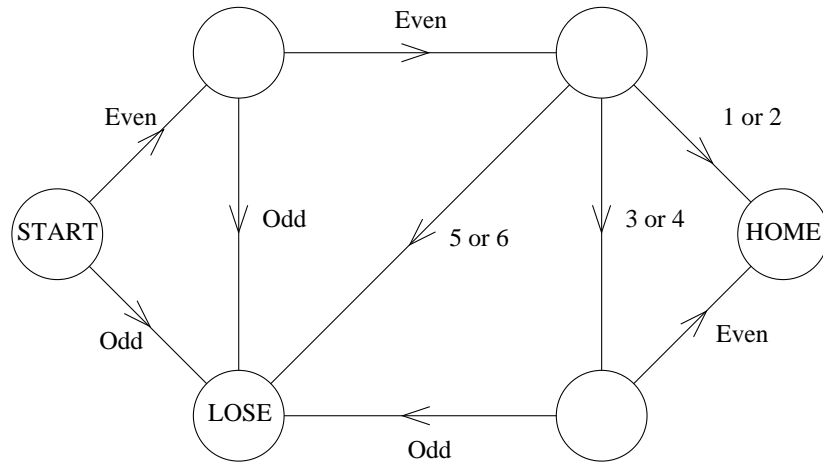
A counter placed on the start circle moves in the direction determined by the throw of a normal six-sided die. What is the probability of reaching the HOME circle?

END OF PROBLEM SET 1