## MATHEMATICAL CHALLENGE 2011-2012

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit. Do not feel that you must hand in answers to all the questions.

## CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE <br> The Edinburgh Mathematical Society, Professor L E Fraenkel, The London Mathematical Society and The Scottish International Education Trust.

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## Middle Division: Problems 2

M1. A set of cards, numbered from 1 to 19, are placed face down on a table. Nine players each pick up two cards. The remaining card is then turned over. The player who achieves the highest total with their two cards plus the number on the remaining card is the winner.
Is it possible for all nine players to have the same total?
If so, what can this total be?
Explain your reasoning.

M2. A computer whizz claims that his program has found some numbers which satisfy Fermat's equation $x^{n}+y^{n}=z^{n}$ for a large integer $n$.
He tells his 10 year old brother that

$$
x=31415926536 \quad y=89173261421 \quad z=90354441655
$$

Almost immediately his brother says that there cannot be any value of $n$ which will work for these numbers. The computer whizz checks his program and finds a bug.
How did his brother know there was a bug?

M3.

| 1 | 2 | 3 |
| :---: | :---: | :---: |
| 4 | 5 | 6 |
| 7 | 8 | 9 |

Take a pair of numbers from the numbers in the grid where the first one is immediately below the second, for example 8 and 5 . Use the numbers to form a calculation as shown:

$$
85-58=27
$$

Using another pair, say 4 and 1, we have

$$
41-14=27
$$

Show that the answer is always 27 , and explain why this happens.

M4. Catriona went on a day trip to the land of Silverglade where the money is very colourful. She exchanged her money for 29 knomes which came as one red note, one yellow note, one blue note and one green note. Her guide pointed out that she now had one of each of the notes available in Silverglade but that many shops did not take the green notes.
At a bank, Catriona exchanged her green note for a blue and a yellow note. At the shop next door she bought a potion for 1 knome and, having handed over two yellow notes, she got three red notes in change. Later, she paid for a postcard costing 1 knome with two blue notes and she got three yellow notes in change.
What is the value of each of the four different coloured notes in Silverglade?
M5. Andy is standing at a bus stop near his house. Through a small window, he can see the reflection of a television in a large mirror. The television set is mounted on the same wall of the house as the window and the mirror is on the opposite wall. He also notices that the reflection he sees through the small window is the full width of the TV but no more.
He wonders how wide his neighbour's TV is. But as the house is exactly like his own he can work it out. The small window is 50 cm wide and the room is 4 m deep. Furthermore he is exactly 10 m from the nearest point on the front wall of the house on which the window and the TV are. How does he calulate this and what is the width of the TV?

END OF PROBLEM SET 2

