The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2011−2012

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, Professor L E Fraenkel,
The London Mathematical Society and The Scottish International Education Trust.
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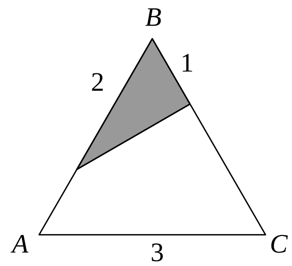
Middle Division: Problems 1

M1. The triangle ABC is equilateral. What fraction of it is shaded?
Explain your reasoning.



M2. Six cousins have a get together and discover that the mean of their ages is 19. Four of the ages are 21, 19, 23 and 11 and range of all six ages is 20. Find the other two ages.
Explain your reasoning.

M3. Andrew set out to cycle from Kirkton to Simsburgh at exactly the same time as Mike left Simsburgh to cycle to Kirkton, along the same route. The two passed each other at the point on the route which is 4 miles from Kirkton. Each cyclist continued to his destination, turned and immediately started cycling back towards his starting point at the same rate. This time, they met 2 miles from Simsburgh.

Assuming that Andrew and Mike travelled at constant speeds, how far is it from Kirkton to Simsburgh? Given that Andrew's speed is v mph, what is Mike's speed?

M4. The rectangular floor of a room is completely tiled with whole tiles, each of which is 15 cm square. The black tiles form a border of width one tile round the room. Within the black border all the tiles are red. There are exactly twice as many red tiles as black ones. Determine the possible lengths and corresponding widths for the room.

M5. Show that there is only one set of different positive integers, x, y, z, such that

$$1 = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}$$

i.e. 1 can be expressed as the sum of the reciprocals of three different positive integers in only one way.

Deduce that, if n is any odd integer greater than 3, then 1 can be expressed as the sum of n reciprocals of different positive integers.

For which even integers is this possible? **Justify your answer**.

END OF PROBLEM SET 1