The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2010−2011

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, Professor L E Fraenkel,
The London Mathematical Society and The Scottish International Education Trust.
The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.
Particular thanks are due to the Universities of Aberdeen, Edinburgh, Glasgow, St Andrews, Strathclyde, and to Preston Lodge High School, Bearsden Academy, Beaconhurst School, St Aloysius College and Turriff Academy.

Middle Division: Problems 2

M1. Four cards, each numbered with a different whole number, are placed face down. Four people, Gavin, Jack, Katie and Luke, in turn select two of these cards, write down their total, and then replace the two cards. Gavin's total is 6, Jack's 9, Katie's 12 and Luke's 15.

Two of the cards are then turned over and their total is 11.

Determine the numbers on each of the cards.

M2. An old-fashioned rectangular billiard table has only four pockets, one at each corner. The lengths of the sides of the table form a whole number ratio.

Show that, if the ratio is 5 : 2 and a ball is hit from one corner at an angle of 45°, it will land in a pocket after 5 rebounds.

If the ratio of the sides were m : n, where m and n are different whole numbers, with no common factor, and the ball were hit from a corner at an angle of 45°, show that the ball would always drop into a pocket after a number of rebounds. How many rebounds would there be in this case?

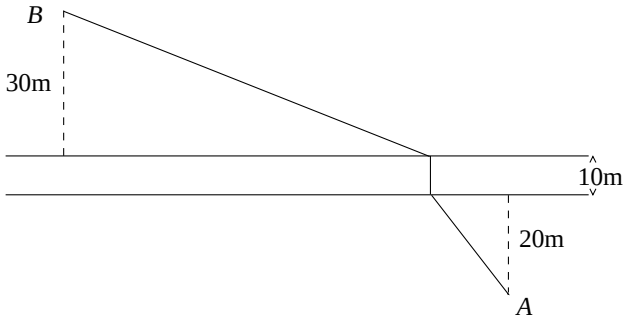
M3. A farmer was having cash-flow problems and was discussing his options with his wife. "If we sell 75 chickens we will bring in some money and my existing stock of feed will last an extra 20 days. But if we buy an additional 100 chickens, we will get money from the extra eggs, but my existing stock of feed will last 15 days less". "Exactly how many chickens do you currently have?" asked his wife.

What is the answer to his wife's question and why is this the answer?

M4. The shape of a fifty-pence piece is based on a regular heptagon which is a 7-sided polygon. The distance between each vertex and each of its two 'nearly opposite' vertices is 1 unit. The perimeter of the coin is formed by circular arcs of radius 1 unit which are centred on each vertex, and join the two nearly opposite vertices. Find the length of the perimeter of the coin.

M5.

A rabbit's burrow is at *A* and he knows that there are carrots in a garden at *B,* across a road, which is 10m wide. The burrow is 20m from the nearer edge of the road and the carrots are 30m beyond the other edge as shown in the diagram. The straight line distance from *A* to *B* is 80m.



The rabbit is wary of crossing the road and knows from past experience that he must cross directly across the road, not askew. What is the length of the shortest possible route for the rabbit from the burrow to the carrots?

END OF PROBLEM SET 2