The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2008−2009

Entries must be the unaided efforts of individual pupils. Solutions must include explanations.

Answers without explanation will be given no credit.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, Professor L E Fraenkel,

The London Mathematical Society and The Scottish International Education Trust.

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Middle Division: Problems 1

M1. On a 26-question test, 8 points were credited for each correct answer and 5 points were deducted for each wrong answer. If all questions were answered, how many were correct if the score was zero?

If Fred and Bernie both scored more than zero, but Fred scored 10 times as many points as Bernie, how many did Fred score correctly and how many did Bernie score correctly? (Again assume that all questions were answered).

M2. Sam had never fully understood the points system in football and felt that the scoring of goals should be encouraged. His idea is that 10 points should be awarded for a win, 5 points for a draw and 1 point for each goal scored, whatever the result of the match. Therefore even if you are losing 0-5 and have no hope of winning, a goal scored might make all the difference between promotion and relegation. This was tried with three teams, Hubs, Dins and Rungs. Each team scored at least one goal in every match and no team played another more than once. Hubs scored 8 points, Dins 14 points and Rungs scored 9 points. Find the score in each match.

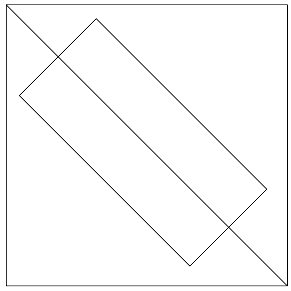
M3. A dishonest market trader has doctored his balance scales so that one arm of the scales is longer than the other, but the pan A on the short side has been made heavier so that a balance is still achieved when the pans are empty. He is selling exotic mixed nuts at £1 for a 250g bag weighed on his scales. He is doing a roaring trade and has emptied seven 24kg sacks of the exotic nuts. This is partly due to his sales gimmick which is, for every 20 bags sold he gives one away “free, gratis and for nothing”. The market regulator finds him out by putting a bag of nuts on pan A of the scales and finding that it balanced with 160g weight on pan B. The trader confesses but pleads that he is a good guy as he has been giving away nuts for free. The regulator agrees to take that into consideration in his calculations, but fines him six times the amount of money he made fraudulently by selling the exotic nuts. How much is the fine?

M4. Three boys went to the local scout jumble sale and visited the book stall. Afterwards they compared their purchases. Harry said to Jake, “ if I give you 6 of my football books for one of your scouting books, you would have twice as many books as I would have”. Then Dai said to Harry, “but if I gave you 14 of my books about cars for one of your scouting books, then you would have three times as many books as I would have”. Finally, Jake said to Dai, “if I gave you 4 of my Beano annuals for one of your scouting books, then you would have six times as many books as I would have”. How many books did each boy have?

If they all had the same number of scouting books and the same number of Beano annuals, and only Harry had football books, how many books of each type did they have? (Assume that they did not have any other types of books.)

M5. Show that if a rectangle, which is twice as long as it is broad, can fit diagonally into a square as shown below, then it can also fit into the square with its sides parallel to the sides of the square.

Is this true if the rectangle is three times as long as it is broad? **Explain your answer**.



END OF PROBLEM SET 1