The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2007−2008

Entries must be the unaided efforts of individual pupils. Solutions must include explanations.

Answers without explanation will be given no credit.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, Professor L E Fraenkel,

The London Mathematical Society and The Scottish International Education Trust.

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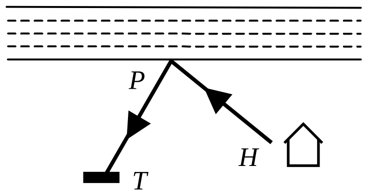
Middle Division: Problems 2

M1. In the village of Piffle, some of the animals are really strange. Ten percent of the dogs think they are cats and ten percent of the cats think they are dogs. All the other cats and dogs are perfectly normal. One day the official village animal psychologist tested all the cats and dogs in the village and found that twenty percent of them thought that they were cats. What percentage of them really were cats?

M2. Every morning, Ruth goes out to take water to her pet armadillo. She picks up her water bucket by her house at the point H on the diagram below, goes down to the edge of the straight river, fills the bucket with water and takes it to the armadillo's trough at T. She has been doing this so often that she knows exactly where the point P on the river bank is, so that she walks the shortest distance. Explain how you should choose the point P on the river bank so that the distance from H to T via P is shortest.



M3. A boy accidentally made an inky mark on the rim of an antique circular table. He tried to hide it by pushing the table into the corner of a rectangular room so that it was touching both walls. When his father came home, he discovered the mark but realised it could be cleaned off. However, pointing to the mark on the far side of the table, he set his son the following problem: he said “That mark is exactly 8 inches from one wall and 9 inches from the other wall. Without measuring it, what is the diameter of the table?”

M4. In a far off country, Tanika's brother gets a weekly allowance of \$13. Being mathematicians, her parents use some special dice to work out how much Tanika should get each week. The dice are fair (i.e. unbiased) and each has six faces with a number on it. The numbers on each die are: 1, 5, 5, 10, 10 and 20. Tanika's allowance is determined by rolling two such dice and taking the sum, in dollars, of the numbers rolled. On average, will she get more or less than her brother and by how much? **Explain your answer.**

M5. A cyclist and a runner start off simultaneously around a racetrack each going at a constant speed in the same direction. The cyclist completes one lap and then catches up with the runner. Instantly the cyclist turns around and heads back at the same speed to the starting point where he meets the runner who has just finished his first lap. Find the ratio of their speeds.

END OF PROBLEM SET 2