The Scottish Mathematical Council

MATHEMATICAL CHALLENGE 2007–2008

Entries must be the unaided efforts of individual pupils. Solutions must include explanations.
Answers without explanation will be given no credit.
CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, Professor L E Fraenkel,
**The London Mathematical Society and The Scottish International Education Trust.**

Middle Division: Problems 1

M1. Find all the six-digit numbers using each of the digits 1, 2, 3, 4, 5, 6 such that the numbers formed
> by the first digit is divisible by 1,
> by the first two digits is divisible by 2,
> by the first three digits is divisible by 3,
> by the first four digits is divisible by 4,
> by the first five digits is divisible by 5,
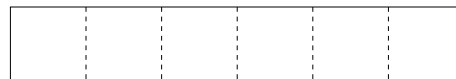> by the first six digits is divisible by 6.

Explain your reasoning.

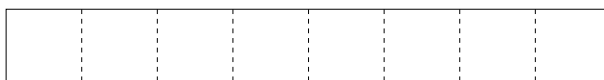**M2.** In this question, you are only allowed to shade complete squares.

(a)   In how many different ways is it possible to shade one half of this rectangle?
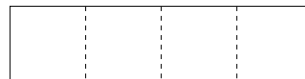
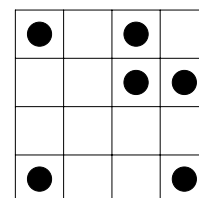(b)   In how many different ways is it possible to shade one third of this rectangle?

(c)   In how many different ways is it possible to shade one quarter of this rectangle?

(d)   In how many different ways is it possible to shade one fortieth of a rectangle made up of 80 squares?

M3. The diagram shows a 4×4 grid containing 6 black spots. These 6 spots are so placed that no three of them lie in a line, either horizontally, vertically or diagonally, but if you add one more spot, there will always be such a line of three spots.
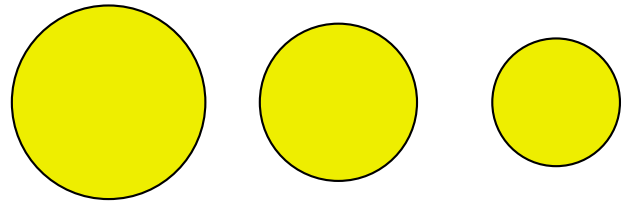What is the largest number of black spots you can place on such a 4×4 grid with this property i.e. no three spots are in a line but if you add any one spot there will always be a line of three?
What is the smallest number of black spots you can place on such a 4×4 grid with this property?
Explain your answers.

M4. Calum was floating down river on a raft, when, half a mile downstream, his brother Duncan set off in a canoe. Duncan paddled downstream as quickly as he could, then turned round and paddled back again, still at his best pace. He and arrived back at his starting point just as Calum floated by.
Assuming Duncan's best pace in still water is 10 times that of the river current, how far did he paddle?

M5. Three large pancakes, as shown, each of the same thickness are to be shared equally among four people. The diameters of the pancakes form a Pythagorean triple. Without measuring, show how to cut the pancakes to make a total of just five pieces so that each person can get the same amount.
Explain your reasoning.

END OF PROBLEM SET 1