J1.
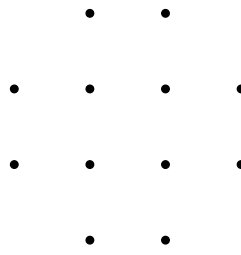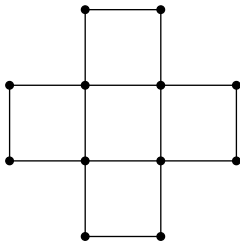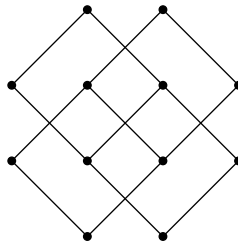
Twelve points are marked on a square grid, as shown. How many squares can be formed by joining four of these points?
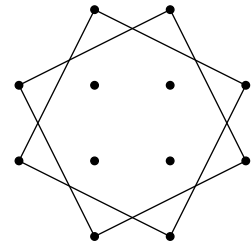
Solution

5 small squares   4 medium squares   2 large squares

Total 11 squares.

J2. The product of the digits of the number 318 is 24. How many 3-digit numbers have digits with product 24?

Solution

$$24 = 1 \times 2 \times 2 \times 2 \times 3$$

(a) If 1 is a digit then the other groups could be 2 and 12; 4 and 6; 8 and 3 but 12 has 2 digits and is ruled out.

| So possible digits are: | 1-4-6 | giving | 146, 164, 416, 614, 461, 641 |
| and | 1-8-3 | giving | 183, 138, 813, 318, 831, 381 |

(b) If 2 is a digit then the other groups could be 2 and 6; 4 and 3.

| So possible digits are: | 2-2-6 | giving | 226, 262, 622 |
| and | 2-4-3 | giving | 243, 234, 423, 324, 432, 342 |

There cannot be any other groups so there are a total of 21 different 3-digit numbers with digits with a product 24.

J3. Water weighs 1000 grams per litre and ether weighs 716 grams per litre. Four litres of a mixture of the two weigh 3.29 kg.

How much water does the mixture contain?

Solution

Let x be the volume of water in litres and y be the volume of ether in litres.

So $x + y = 4$ and $1000x + 716y = 3290$.

From $x + y = 4$ we see that $y = 4 - x$ which means

$$1000x + 716(4 - x) = 3290$$
$$1000x + 2864 - 716x = 3290$$
$$284x = 426$$
$$x = 1.5$$

There are 1.5 litres of water in the mixture.

J4. Imran bought a cat and dog for £60 each. Later he sold them. He made a profit of 20% on the dog. He made a loss of 20% on the cat.

How much did Imran get altogether when he sold the cat and dog?

Later Andy bought another cat and dog. He sold them for £60 each. He made a profit of 20% on the dog. He made a loss of 20% on the cat.

Did Andy make a profit or loss on the whole deal? If so, how much?
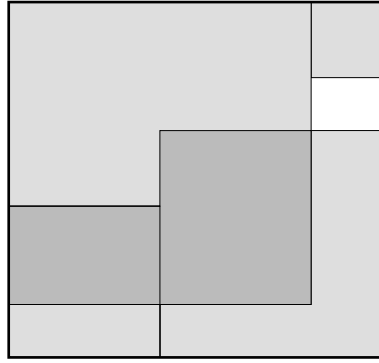
Solution

The 20% of £60 gain exactly balances the 20% of £60 loss, so Imran gets £120 exactly.

Andy sold his dog for £60, which was 120% = 6/5 of what he paid. So he paid 5/6 of £60 = £50.
Andy sold his cat for £60, which was 80% = 4/5 of what he paid. So he paid 5/4 of £60 = £75.

So Andy paid out £125 but only received £120, i.e. he made a loss of £5.

J5.

Four square sheets of tinted glass with sides 10 cm, 20 cm, 30 cm and 40 cm are placed in a rectangular box as shown. 70 square cm of the base of the box are left uncovered (white in the diagram).

What is the area of overlap of the sheets of glass (dark grey in the diagram)?

Solution

The smallest square is 10cm by 10cm so the longer side of the white area is 10 cm. So, as its area is 70 cm^2, its shorter side is $\dfrac{70}{10}$ = 7 cm.

So the top edge in the diagram is 40 cm +10 cm (= 50 cm) (or the same from the bottom edge).

The side on the right is $(10 + 7 + 30)$ cm = 47 cm.

So the area of the whole rectangle is 50 cm $\times$ 47 cm = 2350 cm^2 of which 2350 $-$ 70 = 2280 cm^2 is covered by at least one layer of glass.

But the total area of the glass is
$$40 \times 40 + 30 \times 30 + 20 \times 20 + 10 \times 10 = 3000 \text{ cm}^2.$$

So 3000 $-$ 2280 = 720 cm^2 must be in a double layer.