2020-2021 Junior Division: Problems 1: Solutions

J1. The owner of some stables has to fill in yet another form and so he writes:

"My herd consists of horses and foals.
A fifth of the herd is in the yard and a third of the herd is out to pasture. These are all horses.
Three times the difference are the foals, which are in the barn. The remaining two horses, in the paddock, belong to my daughter."

How many animals are in the herd altogether?

Solution 1

Let there be x animals. The difference between those in the barn and those out to pasture is $\frac{1}{3}x - \frac{1}{5}x = \frac{2}{15}x$ so there are $\frac{6}{15}x$ foals.

Total number of animals:

$$\begin{aligned} \frac{1}{3}x + \frac{1}{5}x + \frac{6}{15}x + 2 &= x \\ \frac{14}{15}x + 2 &= x \\ \frac{1}{15}x &= 2 \\ x &= 30 \end{aligned}$$

There are 30 animals altogether.

Solution 2

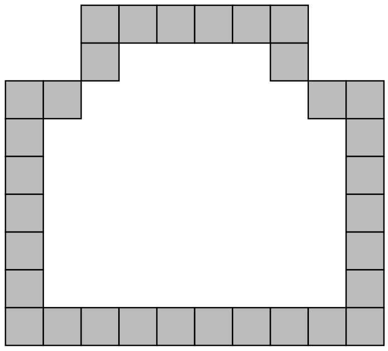
A fifth are in the yard and a third are in the pasture
Difference is 1/3 – 1/5 = 2/15
Foals in barn are three times the difference which is 6/15 = 2/5
So pasture, yard and barn are 1/5 + 1/3 + 2/5 = 3/15 + 5/15 + 6/15 = 14/15
This means that the two horses in the paddock are 1 – 14/15 =1/15 of the total
So the total number of animals is 2 × 15 = 30

Solution 3

The answer must divide by 3 and 5 so it must be a multiple of 15.
If the answer is 15, then 3 are in the yard and 5 are in the pasture so there are 3 × (5 – 3) = 6 foals
Total in yard, pasture and barn is 3 + 5 + 6 = 14, leaving only one in the paddock, so 15 is not the answer.
If the answer is 30, then 6 are in the yard and 10 are in the pasture so there are 3 × (10 – 6) = 12 foals
Total in yard, pasture and barn is 6 + 10 + 12 = 28, leaving two in the paddock, so answer could be 30.
I can see now that if it's more than 30, then too many will be left in the paddock, so 30 is the only possible answer.

J2. The floor of a rectangular room is covered with square tiles. The room is 10 tiles long and 7 tiles wide. In addition, there is a bay window in the middle of a long wall of the room which is 2 tiles deep and 6 tiles across. How many of the tiles have at least one edge which touches a wall of the room?

Solution



In the diagram, the tiles which have at least one edge which touches a wall of the room are shaded grey.
There are 7 tiles along each short edge of the room.
Also 8 *extra* tiles along the long straight wall.
And 10 extra tiles along the wall with the bay window.
So there are $7 \times 2 + 8 + 10 = 32$ tiles which have at least one edge which touches a wall of the room.

J3. Hanna scored an average of 69% on 7 tests. If the lowest scoring test is omitted, what is her highest possible average on the remaining six tests?

Solution
To make the average on the remaining tests as high as possible the omitted score needs to be as low as possible.
The lowest possible score on the omitted test is 0.
The total score of 69×7 is now averaged over 6 tests: average = $\dfrac{69 \times 7}{6} = 80.5$
So the highest possible average on the remaining six tests is 80.5%.

J4. Jo is going on an 8-day activity holiday. Each day she can choose one of the water sports: kayaking or sailing, or land-based sports. She never does different water sports on consecutive days. She also wants to try all three options on at least one day of her holiday.

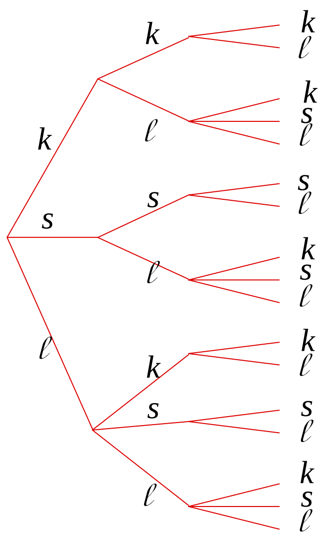
How many different schedules are possible?

Solution

On the first day all 3 choices are possible.

After kayaking, only 2 choices are possible: kayaking or land-based. After sailing, only 2 choices are possible: sailing or land-based.

After land-based, all 3 choices are possible.



So we build up a tree diagram:

Day 1 choices: $k + s + \ell = 3$

Day 2 choices: $2k + 2s + 3\ell = 7$

Day 3 choices: $5k + 5s + 7\ell = 17$

Day 4 choices: $12k + 12s + 17\ell = 41$

Day 5 choices: $29k + 29s + 41\ell = 99$

Day 6 choices: $70k + 70s + 99\ell = 239$

Day 7 choices: $169k + 169s + 239\ell = 577$

Day 8 choices: $408k + 408s + 577\ell = 1393$

Note that this total includes holidays which have only one or two different activities.

It is not possible to have a schedule with only kayaking and sailing.

With only sailing and land-based there are $2^8 = 256$ possible schedules.

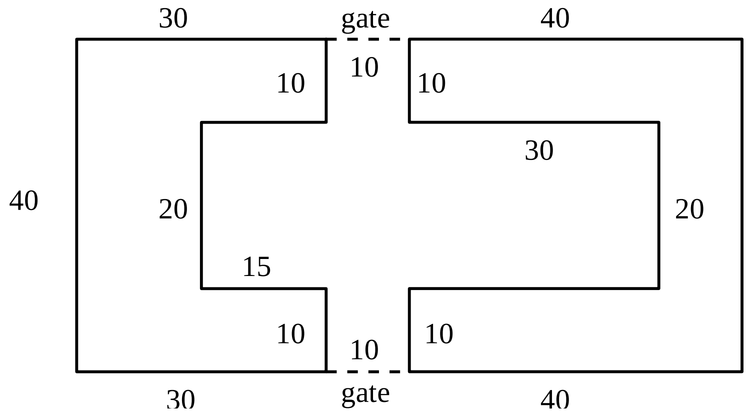
With only kayaking and land-based there are also $2^8 = 256$ possible schedules.

The schedule with only land-based is in both groups.

So there are $256 \times 2 - 1 = 511$ schedules with 2 or 1 different activities.

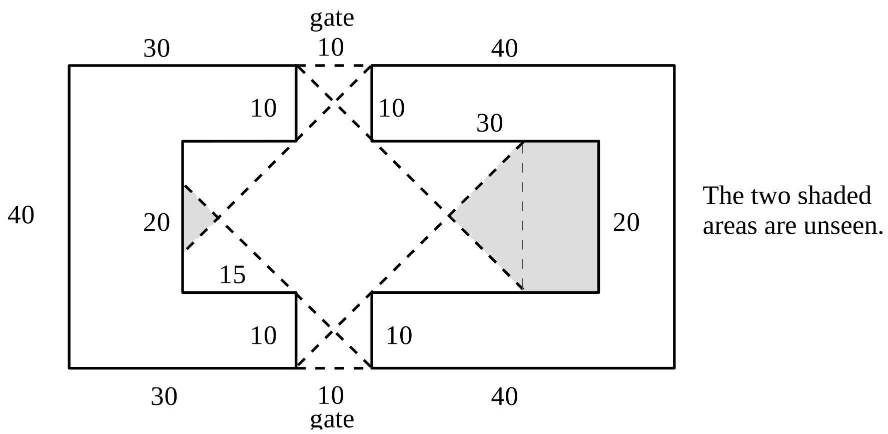
So there are $1393 - 511 = 882$ schedules which include all 3 activities.

J5.



This is the plan of a building which has a courtyard with two entrance gates. Passers-by can look in through the gates but may not enter. The dimensions are given in metres and all corners are right angles. What is the area of the part of the courtyard which cannot be seen by passers-by?

Solution



Left hand unseen area:

Square with diagonal 10 m has side $\frac{10}{\sqrt{2}}$ m and area 50 m^2

So left hand unseen triangle has area 25 m^2.

Right hand unseen area:

Square with diagonal 20 m has side $\frac{20}{\sqrt{2}}$ m and area 200 m^2.

So right hand unseen triangle has area 100 m^2 and the right hand unseen rectangle has area 200 m^2.

Total unseen area is 325 m^2.