Junior Division: Solutions to Problems 2

J1. Three buckets are coloured red, green and blue. Each bucket contains four balls numbered 1, 2, 3, and 4, of the same colour as the bucket. Without looking, Emily chooses one ball at random from each of the buckets. If r, g and b are the numbers on the balls chosen from the red, green and blue buckets respectively, Emily wins a prize when $r = g + b$.

What is the probability that Emily wins a prize?

Solution

There are $4 \times 4 \times 4 = 64$ possible different, equally likely choices of the three balls.

Think about the green and blue choices that could win:

g	b
1	1
1	2
2	1
2	2
3	1
1	3

All other choices have a total of more than 4 and so lose.

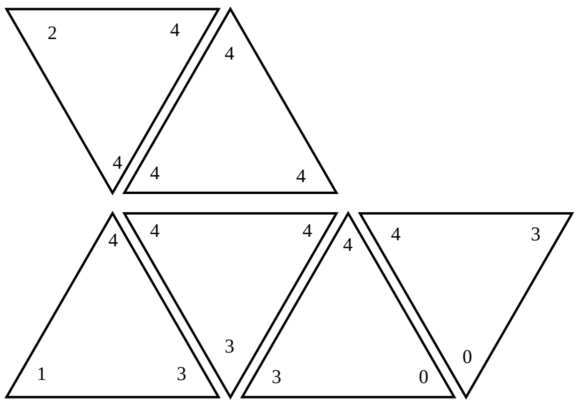
In each case, only one of the 4 red balls will give the required total.

So there are 6 ways to win out of the 64 possible choices.

Emily wins a prize with probability $\frac{6}{64} = \frac{3}{32}$.

J2. Three-sided dominoes are equilateral triangles and one face of each domino has a number in each corner and the other side is blank. The numbers range from 0 up to the highest number in the set, 4. Here is an example of a game which started with the 4 4 4 domino.

The set contains all possible different dominoes. How many dominoes are there in the set?



Solution

List the dominoes systematically, starting with the one with 0 in each corner, then adding those with at least one 1, then those with at least one 2 and so on. Dominoes with three different numbers have a mirror image − see the 4 3 0 domino in the example game.

0 0 0 i.e. 1 way of creating dominoes with numbers up to 0.

1 0 0
1 1 0
1 1 1 i.e. 3 ways of creating dominoes with a 1 and other numbers not greater than 1.

2 0 0
2 1 0
2 0 1 (a mirror image of the previous one)
2 1 1
2 2 0
2 2 1
2 2 2 i.e. 7 ways of creating dominoes with a 2 and other numbers not greater than 2.

3 1 0
3 0 1
3 2 0
3 0 2
3 2 1
3 1 2 (3 ways of choosing the first different number after a 3, and 2 ways of choosing the last different number i.e. 6 dominoes in all.)

3 0 0
3 1 1
3 2 2 (3 ways of choosing a repeated number after a 3)

3 3 0
3 3 1
3 3 2 (3 ways of choosing a number after a repeated 3)

3 3 3 (1 way)
i.e. 6 + 3 + 3 + 1 = 13 ways of creating dominoes with a 3 and other numbers not greater than 3.

4 ways of choosing the first different number after a 4, and 3 ways of choosing the last different number i.e. $4 \times 3 = 12$ dominoes in all.
4 ways of choosing a repeated number after a 4
4 ways of choosing a number after a repeated 4
1 way of choosing 4 4 4
i.e. $12 + 4 + 4 + 1 = 21$

So there are 1 + 3 + 7 + 13 + 21 = 45 ways of creating dominoes with a 4.
There are 45 3-sided dominoes in the set.

J3. (In this question, the possible coins are: 1p, 2p, 5p, 10p, 20p, 50p, £1 and £2.)

You and a friend have been saving for your holidays, but you still have a long way to go. So far, you have both saved the same amount of money: £15.39. You discover that your friend has four different types of coin and has the same number of each type. This surprises you, because you have four different types of coin and the same number of each type.

Do you have the same number of coins as your friend does?

Do not forget to give reasons for your answer.

Solution

The total sum of money is 1539p. Since there are the same numbers of coins of each type, the number of coins is a factor of 1539.

When we factorise 1539 we get: $1539 = 171 \times 9 = 81 \times 19$. So the possible factors are:

$3 \times 513 \qquad 9 \times 171 \qquad 27 \times 57 \qquad 81 \times 19$

If we can make two different factors each from 4 different coins then we will know that you and your friend may have different numbers of coins

$81 = 50 + 20 + 10 + 1 \qquad 171 = 100 + 50 + 20 + 1$

So there *are* two possible ways to make £15.39:

19 sets of 50p, 20p, 10p, and 1p using 76 coins in all,

or

9 sets of £1, 50p, 20p and 1p using 36 coins in all.

So you and your friend may have the same or different numbers of coins.

J4. How many positive multiples of 7 that are less than 1,000 end with the digit 3?

How many positive multiples of 7 that are less than 10,000 end with the digits 33?

Solution

If a multiple of 7 ends in a 3, the next multiple of 7 ends in a 0. As they also have to be multiples of 7, these are 70, 140, 210, ..., 980. There are 14 of these so there are 14 multiples of 7 less than 1000 which end with the digit 3.

If a multiple of 7 ends in 33, the next multiple of 7 ends in 40. These must also be such that when they are divided by 10 they produce a multiple of 7. These are 140, 840, 1540, 2240, 2940, 3640, 4340, 5040, 5740, 6440, 7140, 7840, 8540, 9240, 9940. So we can say there are 15 positive multiples of 7 that are less than 10 000 end with the digits 33. They are each of the numbers 140, 840, ... reduced by 7.

J5. A cross-country skier practices the same route from his home to his friend's home and leaves at the same time every day. He realised that when he skis at 10 mph he arrives at 4 minutes past noon, and when he skis at 15 mph he arrives at 4 minutes before noon. How fast would he have to go to reach his friend's house at noon exactly?

Solution

Let the distance be d miles. Then the time taken when skiing at 10mph is $\frac{d}{10}$ hours and the time taken when skiing at 15 mph is $\frac{d}{15}$ hours.

So, we have

$$\begin{aligned} \frac{d}{10} &= \frac{d}{15} + \frac{8}{60} \\ 6d &= 4d + 8 \\ 2d &= 8 \\ d &= 4 \end{aligned}$$

The distance is 4 miles.

To cover 4 miles at 10 mph would take 0.4 of an hour which is 24 minutes. As he arrives at 12.04 skiing at 10 mph, it means he sets off at 11.40.

So to reach his friend's house at noon the time would be 20 minutes and the speed would be $\frac{4}{\frac{1}{3}} = 4 \times \frac{3}{1} = 12$,
ie the skier will have to travel at 12 mph.