2019-2020 Junior Division: Problems 1 solutions

J1. A farmer has packed several baskets either with chicken eggs or with duck eggs. The number of eggs in each basket is 5, 6, 12, 14, 23 and 29. Her daughter says "If we sell one basket then we will have twice as many chicken eggs as duck eggs left".
Which basket was the daughter thinking about?

Solution

The number of eggs in each basket is 5, 6, 12, 14, 23 and 29 but we don't know which basket has chicken eggs and which has duck eggs.
If we remove one basket then the remaining numbers must split into twice as many chicken eggs as duck eggs ie the total would have to be a multiple of 3.

The total number of eggs is 5 + 6 + 12 + 14 + 23 + 29 = 89.
If we remove one basket at a time we get:

89 − 5 = 84	possible
89 − 6 = 83	not possible
89 − 12 = 77	not possible
89 − 14 = 75	possible
89 − 23 = 66	possible
89 − 29 = 60	possible

Out of the four possible options left:

Selling 5 leaves 84 split into 56 chicken eggs and 28 duck eggs: the only way to make 28 is with 23 + 5 but we don't have the basket with 5 eggs so not possible.

Selling 14 leaves 75 split into 50 chicken eggs and 25 duck eggs: 14 + 5 + 6 = 25 but we no longer have the basket with 14 so not possible.

Selling 23 leaves 66 split into 44 chicken eggs and 22 duck eggs: not possible to make the total 22.

Selling 29 leaves 60 split into 40 chicken eggs and 20 duck eggs: 14 + 6 = 20 duck eggs and 23 + 12 + 5 = 40 chicken eggs.

So the basket sold was the one with 29 eggs.

J2. Three rugs have a combined area of 200 m^2. By overlapping the rugs to cover a floor area of 140 m^2, the area which is covered by exactly two layers of rug is 24 m^2. What is the area of floor that is covered by three layers of rug?

Solution

The extra amount of rug laid on top of the first layer is 200 − 140 = 60 m^2.
Of this extra rug, 24 m^2 is laid in exactly two layers. So there are 60 − 24 = 36 m^2 of extra rug laid in the second and third layers i.e. 18 m^2 in each layer.
Hence 18 m^2 of floor is covered by exactly 3 layers.

J3. A cup of coffee costs more than £1 but less than half the cost of a piece of cake. Two cups of coffee and seven pieces of cake cost £18.27. How much could a cup of coffee cost?
(The prices are exact whole numbers of pence.)

Solution
Let x pence be the cost of a cup of coffee and y pence the cost of a piece of cake. So we know

$$x > 100 \qquad\qquad x < \tfrac{1}{2}y \qquad\qquad 2x + 7y = 1827$$

Note that $1827 = 7 \times 261$ so we can rewrite the third condition as $\frac{2}{7} + y = 261$.

We also know from this that x is divisible by 7 so we start with the smallest multiple of 7 which exceeds 100 and see what happens!

x	$\frac{1}{7}x$	$\frac{2}{7}x$	y	$\frac{1}{7}y$	Is $x < \frac{1}{2}y$?
105	15	30	231	$115\frac{1}{2}$	Y
112	16	32	229	$114\frac{1}{2}$	Y
119	17	34	227	$113\frac{1}{2}$	Y

So when $x = 105$, we get $y = 231$ and when $x = 112$, we get $y = 229$. So we know that a cup of coffee costs either £1.05 or £1.12.

J4. More than 10 but fewer than 30 hikers set out on a walk. When they stopped for lunch the party decided to split, one group taking a shorter route back. Harry led the group taking the shortcut and Mac led the other.

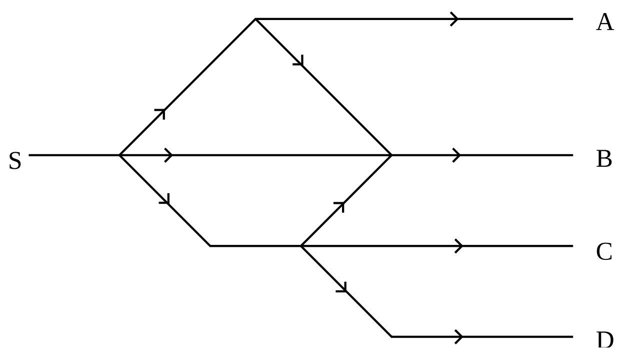
Each hiker chose their group, but during lunch Colin changed his mind and decided to join Harry's group. This gave the same number in each group. However Colin began to feel much stronger after his lunch and decided to go with Mac after all, and Dave also decided to push on with Mac. The number of hikers in each group was now a prime number.

How many hikers returned in each group?

Solution
Suppose that there were h in Harry's group and m in Mac's group at the start. So after Colin changed, $h + 1 = m - 1$, and therefore $m = h + 2$.
From the next moves, we know that $h - 1$ and $m + 1$, which is the same as $h + 3$, are both primes.

The only primes which allow between 10 and 30 on the walk and differ by 4 are 7 and 11.
There were 7 in Harry's group and 11 in Mac's.

J5. Rhoda Rat is put in a maze at the start, S. She can move forward only in the direction of the arrows. At each junction she is equally likely to choose any of the forward paths.
What is the probability that she ends up at B?



Solution
At the three-way junctions Rhoda has a $\frac{1}{3}$ chance of taking each path and at the two-way junction she a $\frac{1}{2}$ chance of taking each path.
So her probability of arriving at B is

$$\left(\frac{1}{2} \text{ of } \frac{1}{3}\right) + \frac{1}{3} + \left(\frac{1}{3} \text{ of } \frac{1}{3}\right) = \frac{1}{6} + \frac{1}{3} + \frac{1}{9} = \frac{3 + 6 + 2}{18} = \frac{11}{18}.$$