2018-2019 Junior Division: Solutions to Problems 2

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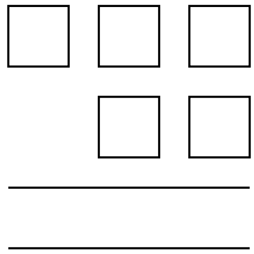
J1. Each of the digits 2, 3, 5, 7 and 8 is placed one to a box in the diagram.



(a) If the two-digit number is subtracted from the three digit number, what is the smallest possible difference?

(b) If the three-digit number is multiplied by the two-digit number, what is the smallest possible product?

Solution

(a) To make the difference small, we need the *smallest* possible 3-digit number and the *largest* possible 2-digit number.

In the 3-digit number, choose the hundreds first, then the tens and finally the units. So the smallest number of hundreds is 2, the smallest number of tens is 3 and the smallest number of units is 5. The number is 235.

In the 2 digit number, choose the tens first and then the units. So the largest number of tens is 8 and we are left with 7 units. The number is 87.

The smallest difference is then $235 - 87 = 148$.

(b)(i) In each number the digits have to be in **ascending** order, because this will give you smaller numbers and therefore a smaller product. The possibilities are

Two-digit numbers starting with 2	Two-digit numbers starting with 3
$23 \times 578 > 10000$	$35 \times 278 = 9730$
$25 \times 378 = 9450$	$37 \times 258 = 9546$
$27 \times 358 = 9666$	$38 \times 257 = 9766$
$28 \times 357 = 9996$	

Where the two-digit begins with 5 or 7 the products all exceed 10000.

So the smallest product is $25 \times 378 = 9450$.

(b)(ii) At first sight there seem to be a lot of products to be done. There are 5 choices for the digit which goes in the left-hand box in the top row. There are then 4 choices for the digit which goes in the middle box in the top row and then 3 choices for the digit which goes in the right-hand box in the top row. There are then 2 choices for the digit that goes in the left-hand box in the second row and then there is only 1 choice left for the last box. So the number of ways of filling all five boxes is $5 \times 4 \times 3 \times 2 \times 1$, which gives 120 different products. Fortunately we shall only need to do 6 out of the 120, as we shall now show.

Suppose we put the digits 3, 5 and 7 in the top row in some order and put 2 and 8 in the second row in some order. One possible product would then be 537×82. However it is easy to see that the product which gives the smallest answer is 357×28 where in each of the two factors the digits appear in increasing order. This reduces the number of products to be examined to the following 10, which we can list in increasing order of their two-digit factors

(a) 578×23	(b) 378×25	(c) 358×27	(d) 357×28	(e) 278×35
(f) 258×37	(g) 257×38	(h) 238×57	(i) 237×58	(j) 235×78.

Without working out the exact value, we can see that (a) is bigger than $500 \times 200 = 10000$. We can obtain similar results for (h), (i) and (j). On the other hand, the product in (b) can be written as $(378/4) \times 100 = 9450$. So we can discard (a), (h), (i) and (j). That leave us with five products to work out. The answers are

(c) 9666	(d) 9996	(e) 9730	(f) 9546	(g) 9766

Hence the smallest possible product is 9450.

J2. A jeweller makes sets of small cubes out of solid silver. The jeweller has gold-plated none, some, or all of the faces on some of the cubes. The cubes in a set are all different, and no other cube can be added to the set. How many cubes are there in a set?

Solution

Each cube has 6 faces so we need to identify the number of different cubes with each of 0, 1, 2, 3, 4, 5 or 6 faces gold-plated

Number of faces gold-plated	Number of different cubes
0	1
1	1
2	1 with an edge in common
	1 which opposite faces plated
3	1 where all 3 faces have a common corner
	1 where 3 faces are U-shaped
4 (the opposite of 2 faces)	1 and 1
5 (the opposite of 1 face)	1
6 (the opposite of 0 faces)	1

So there are 10 cubes in the set.

J3. Near where I live, there is a very short street of 14 houses numbered 1 to 14 – seven on each side with the odd numbers on one side and the even numbers on the other (numbers 1 and 2 face each other). The really interesting thing about the street is that all the people along one side have names that sound like trades or crafts, while all those on the other side have names which sound like colours.

- Mr Fletcher and Mr Wright live respectively opposite Mr Green and Mr White, who are both neighbours of Mr Black.
- Mr Smith is Mr Mason's father-in-law.
- Mr Mason lives in a higher number than Mr Brown. Mr Mason's and Mr Brown's numbers together equal those of Mr White and Mr Wright together.
- Mrs Taylor's number is twice that of her sister, Mrs Tyler.
- Mr Gray lives opposite to Mr Baker.
- Mrs Tann lives in a double-figure number opposite to her daughter, Mrs Taylor.

What is Mr Scarlett's number?

Solution

1	3	5	7	9	11	13
2	4	6	8	10	12	14

'Mrs Taylor's number is twice that of her sister, Mrs Tyler.'
This means that the trades must be in the even numbers and the colours in the odd numbers.

'Mrs Tann lives in a double-figure number opposite to her daughter, Mrs Taylor.'
Mrs Tann must be in number 11 or 13. Then Mrs Taylor would be in 12 or 14, but then Mrs Tyler is in 6 or 7. Mrs Tyler (trade) cannot be in an odd number so
Mrs Tyler – 6; Mrs Taylor – 12 and Mrs Tann – 11.

1	3	5	7	9	11 Tann	13
2	4	6 Tyler	8	10	12 Taylor	14

‘Fletcher and Wright live respectively opposite Green and White, who are both neighbours of Black.’

This means that Green, Black and White are next to each other: possible numbers are 1,3,5; 3,5,7; or 5,7,9. The first and last of these options is not possible because Tyler would be where Fletcher or Wright should be so it must 3,5 and 7. It could be Green, Black and White or White, Black and Green.

1	3 Green or White	5 Black	7 White or Green	9	11 Tann	13
2	4 Fletcher or Wright	6 Tyler	8 Wright or Fletcher	10	12 Taylor	14

‘Mason lives in a higher number than Brown. Mason’s and Brown’s numbers together equal those of White and Wright together.’

Mason must be in 2, 10 or 14 and Brown 1, 9 or 13 giving possible totals of 3, 11, 15, 19, 23 or 27.

White and Wright total must be 7 or 15.

Mason – 14; Brown – 1; White – 7 and Wright – 8.

1 Brown	3 Green	5 Black	7 White	9	11 Tann	13
2	4 Fletcher	6 Tyler	8 Wright	10	12 Taylor	14 Mason

‘Gray lives opposite to Baker.’

Only position available is Gray – 9 and Baker – 10. Leaving Smith – 2 and Scarlett – 13.

1 Brown	3 Green	5 Black	7 White	9 Gray	11 Tann	13 Scarlett
2 Smith	4 Fletcher	6 Tyler	8 Wright	10 Baker	12 Taylor	14 Mason

Scarlett lives in number 13.

J4. An unlimited supply of petrol is available from a camp at one edge of a desert which is 800 miles wide but no petrol is available anywhere else. A truck can only carry enough petrol to travel 500 miles and is able to leave petrol to be collected later. (There is no limit on the size of such stocks and it should be assumed that no petrol is lost by evaporation or spillage.) Establish whether or not it is possible for the truck to get across the desert and, if it is, explain how.

Solution

Working backwards:

(a) With just 500 units of petrol, the lorry can go 500 miles.

(b) With 1000 units of petrol, the lorry fills up; goes $500 \div 3$ miles, leaves $500 \div 3$ units of petrol in the cache, returns, fills up, drives $500 \div 3$ miles then fills up and can go a further 500 miles. Total distance $500\,(1 + \frac{1}{3})$ miles.

(c) With 1500 units of petrol, the lorry fills up, goes $500 \div 5$ miles, leaves $500 \times (3 \div 5)$ units of petrol, returns and repeats, so that $500 \times (6 \div 5)$ units of petrol have been left in the second cache. The the lorry fills up and returns to the second cache. The lorry uses $1 \div 5$ units of petrol to reach the second cache, so has $500 \times (4 \div 5)$ units of petrol left in the tank. So there are 500×2 (i.e. 1000) units of petrol in the second cache, ready to carry out step (b).

Total distance $500\,(1 + \frac{1}{3} + \frac{1}{5}) < 800$.

(d) We need to extend the range to 800 miles. As $800 = 500\left(1 + \frac{9}{15}\right)$, we need another 500/15 miles. We need to create a third cache of 1500 units of petrol ready for step (c).

The lorry fills up, drives 500/15 miles, leaves $500 \times \frac{13}{15}$ units of petrol, drives back and repeats twice. The third cache now contains $500 \times \frac{39}{15}$ units of petrol. Now the lorry sets off with $500 \times \frac{7}{15}$ units of petrol and drives to the third cache. Here the lorry has $500 \times \frac{7}{15}$ units of petrol in the tank, added to the $500 \times \frac{39}{15}$ units of petrol already there makes 1500 units of petrol, ready to carry out step (c).

Total distance $500\,(1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{15}) = 500\left(1 + \frac{9}{15}\right) = 800$ miles as required.

Alternatively, the third cache could follow the pattern above at $500 \div 7$ miles. Then the lorry would arrive with some petrol left in the tank.

J5. Two ships, one 200 metres in length and the other 100 metres in length, travel at constant but different speeds. When travelling in opposite directions, it takes 20 seconds for them to completely pass each other. When travelling in the same direction, it takes 50 seconds for them to completely pass each other.

Find the speed of the faster ship.

Solution

Let the speed of the faster ship be v metres per second and the speed of the slower ship be u metres per second.

To pass, the faster ship must travel the length of the slower ship plus its own length, a total of

$$100 + 200 = 300 \text{ metres.}$$

When the ships travel in the same direction the relative speed is $v - u$ metres per second. So

$$v - u = \frac{300}{50} = 6.$$

When the ships travel in opposite directions the relative speed is $v + u$ metres per second. So

$$v + u = \frac{300}{20} = 15.$$

Hence $2v = 21$ so $v = 10.5$.

The speed of the faster ship is 10.5 metres per second.